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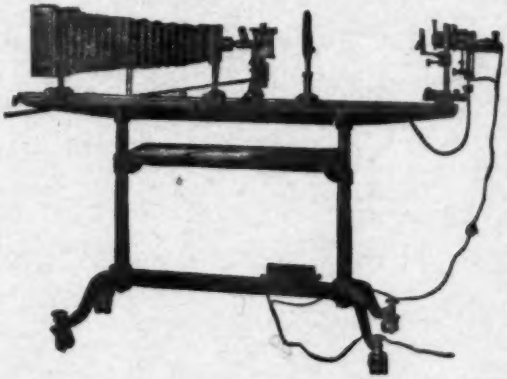

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Vol. VI AUGUST, 1922 Number 6

REPORT OF COMMITTEE ON COLORIMETRY FOR 1920–21

BY
L. T. TROLAND, CHAIRMAN[1]

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I. INTRODUCTION

That the nomenclature and standards of color science are in an extremely unsatisfactory condition is manifest to practically all workers in this field. It is the purpose of the present report to take an initial step towards remedying this state of affairs. That the result cannot be final as regards either nomenclature or standards is a natural consequence of the pioneer character of the effort.

The terminology which is proposed in the following pages represents an endeavor to crystallize the consensus of usage among experts, but where experts disagree and extant terms are vague, it has been deemed wise to introduce certain innovations. While the recommendations of this report are tentative, it is hoped that their careful consideration will assist in the clarification of ideas and the eventual unification of nomenclature. It is desired that all interested persons present their objections directly to the Committee, with a view to the resolution of possible disagreements. Every relevant idea will thus be thrown into the "melting pot" and the final product should be maximally satisfactory to all concerned.

In addition to its attempt (1) to outline a clear terminology, the following report endeavors (2) to summarize in usable form the best available psychophysical data relating color to its stimulus conditions, (3) to formulate or to define certain standard color stimuli—or intensity distributions of radiant energy (or allied quantities), (4) to outline briefly the principal methods of color measurement and (5) to establish fundamentally the relations between their respective scales. A detailed analysis of the techniques and terminologies of the special methods together with a discussion of the best instruments available, or proposed, for applying them, are reserved for a later report.

The incompleteness and imperfection of the data and methods of color science are only too apparent to the Committee, which

NOTE: The numbers within brackets in the text and footnotes refer to books and papers listed in the appended bibliography. The full face numbers are the serial bibliography numbers while succeeding numbers in ordinary type represent the pages in the given book or article to which reference is specifically made.

recognizes clearly that much must be added and many changes be made before the work can attain its final goal: the production of an authoritative and satisfactory text on colorimetrics. The present draft may serve not only as a presentation of the possessions, but also of the *needs* of the science, and become a stimulus—as well as an aid—to new contributions.

II. NOMENCLATURE

The discussion of the general terminology of colorimetrics may be divided into three sections dealing respectively with (1) psychological terms, (2) stimulus terms, and (3) psychophysical terms. In the present part of the report we shall consider only general conceptions, the detailed terminology and symbolism involved in special methods of color designation being presented—if at all—in connection with the discussion of the several methods.

1. PSYCHOLOGICAL TERMS

A. COLOR.—[2]**Color is the general name for all sensations**[3] **arising from the activity of the retina of the eye and its attached**

[2] The definition of the term *color* which is advocated in the present report is the result of very careful consideration and protracted debate between various members of the Committee. It is unfortunate that in common speech the word color is employed, in different contexts, with at least two different meanings which are mutually inconsistent. The most common usage of the word makes it denote visual qualities which possess hue or have a finite degree of saturation, thus excluding all members of the gray series, including black and white. The second common usage of the word color is in harmony with the one recommended in the present report and causes it to embrace all visual qualities within its meaning. This second usage is most frequently found in the interrogative mood. For example, if we ask, "What is the color of a house?" it is as legitimate an answer to say "white" or "gray" as to say "red" or "green." On the other hand, the statement "the woman wore a colored dress" evidently excludes grays from the intended meaning. Such terms as color-photography, color-blindness, etc., have a similarly restricted meaning.

It is scarcely admissible in a scientific terminology to employ one term in two distinct and closely allied senses, since this will inevitably lead to confusion. Consequently, it is necessary to reject one of the common-speech meanings of the word color. A careful study of the situation shows, however, that the rejection of either meaning must result, in the beginning, in certain perplexities. If we employ color in the broader sense we not only sacrifice a well recognized distinctive term for the hue-saturation aspects of visual experience but we also seem to discard a large number of terms derived from the Greek root *chroma* which have been used in the same sense. On the other hand, if we define color in the restricted sense to exclude the gray series we find it necessary to exclude all considerations of brilliance from the field of colorimetry.

nervous mechanisms, this activity being, in nearly every case in the normal individual, a specific response to radiant energy of certain wave-lengths and intensities. It may be exemplified by an enumeration of characteristic instances, such as red, yellow, blue, black, white, gray, pink, etc.

This means that if we are asked to specify the color of a gray object we must state that it has no color, and hence lies outside of our province. Similarly, we should be compelled to affirm that certain browns are identical in color with certain yellows, oranges, and reds because they possess the same hue and saturation, although their brilliances are quite different. The necessity of reactions of this sort on the part of the scientific colorimetrician would cause serious embarrassment in practice. It seems necessary to permit a certain degree of overlapping of the provinces of colorimetry and photometry, and possibly it would be desirable to include the latter under the former as a special branch.

A way out of this dilemma appears possible to the chairman if we can decide to employ the Greek root, *chroma*, in a different sense from the Latin root, *color*. There seems to be no particular etymological reason for regarding these roots as exact equivalents, and it is in line with economy of terminology to differentiate between their technical meanings. We therefore propose that the root, *color*, and its derivatives be employed hereafter to designate all visual qualities, including those of the gray series as well as those possessing hue and saturation. (The German equivalent, *Farbe*, is already used in this sense.) The root, *chroma*, and its derivatives, on the other hand, will be used to designate visual qualities possessing hue and saturation and excluding the gray series, with its terminal members, black and white. Such a separation of meanings is far more defensible etymologically than many distinctions which have been formally established in scientific nomenclature; for example, the distinction between *physics*, the general science of material properties, and *physiology*, the special science of vital processes, both of which terms must be considered to have the same etymological significance because of the common Greek root which they contain.

In harmony with the above general recommendation the following subsidiary developments may be indicated. The word, *chroma*, may be substituted bodily for the word, color, when the latter is intended in the restricted sense, thus red, green, pink, lavender, etc., are chromas, while black, any gray, and white are not chromas, although all of these qualities are correctly designated as colors. This usage of the term chroma may involve a slight confusion with its use by some authorities as a synonym for saturation but it will be noted that the change involved is only a small one, being simply the substitution of a qualitative for a quantitative meaning in practically the same context. The present report recommends that chroma be not used as an equivalent of saturation. If we recognize the suggested distinction between the Greek and Latin roots it is not a contradiction in terms to speak of an "achromatic color" nor is it a tautology to refer to a "chromatic color." (cf. German: *bunten Farben*.)

The distinction in question has the advantage of preserving all of the derivatives of the root, chroma, in their accepted meanings, and there are so many of these derivatives so firmly fixed in scientific discussion as to make it practically impossible to eliminate or to modify them. All such terms as chromatic, achromatic, chromaphore,

It is imposible to identify color with radiant energy, or with wave-lengths of radiant energy, although radiant energy is the adequate stimulus for color. This is because color is known to depend upon the presence and character of the perceiving individual and because it is directly recognized to be something radically different in kind from its stimuli. Consequently, nothing but confusion can result from the use of the word "color" as a synonym

monochromatic, dichromatic, trichromatic, photochromatic, etc., will be taken to refer to colors possessing hue and saturation or to the stimulus or organic conditions underlying the production of such colors. The root, chroma, and its derivatives provide us with a well established and hence constantly available means for differentiating between color in a restricted sense and members of the gray series, while "color" and its derivatives provide us with a means for designating both of these meanings together.

Some difficulties of course arise and must be met courageously by bold changes in usage. Fortunately the cases in question are not very important. For example, "chromatics" can no longer be regarded as synonymous with "colorimetry," chromatics being strictly the science of hue and saturation coördinate with photometry, if the latter is also regarded as a sub-division of colorimetry. The term "colorless" cannot be regarded as the equivalent of "achromatic" and must be taken to indicate complete transparency as well as achromaticity in an object. This is probably already the most common meaning of the term. The equivalent of the phrase "a colored object" in the common restricted usage of the term color would be "a chromed object." The phrase "color vision" becomes redundant and must be replaced by "chromatic vision." The terms relating to "color-blindness" may need some revision but the most common forms of this disorder are already designated as *partial* color-blindness, a designation quite in harmony with our usage of the term color. However, "total color-blindness" would be the equivalent of "complete blindness" on this basis and hence the word *achromatopia*, already in use, will be necessary in this instance.

It is the opinion of the Committee that the above suggestions, although necessitating a number of radical changes, involve a minimum of such changes among the possibilities which are open to us in improving the nomenclature of color science. However, the recommendations of the present report are intended to be tentative and the Committee will be glad to listen to alternative proposals and to objections to the particular form taken by the present suggestions, which represent a compromise between strongly opposed factions, all well represented in the Committee.

[3] The word sensation is used here to stand for an elementary form of experience or consciousness normally depending upon the operation of a sense organ. Although the existence of any sensation rests upon the operation of the nervous system, this should not lead us to *localize* it in that system. Although color is not a physical entity, it obviously exists outside of us on the surfaces of objects as we see them, such visual objects or perceptions being themselves nothing but arrangements of color areas in space. This statement, however, should not be misinterpreted to mean that the colors are physical or are located on physical objects. There is no reason for supposing that visual objects are identical or coincident with the objects of physical science.

of "wave-length" or "wave-length constitution."[4] Color cannot be identified with or reduced to terms of any purely physical conception; it is fundamentally a psychological category.[5]

B. THE THREE ATTRIBUTES OF COLOR.—The nature of any color can be completely specified psychologically in terms of three fundamental attributes, this specification taking the form of an immediate description of the color, as such, *without any reference whatsoever to the stimulus*. The names employed for these three attributes by different authorities vary widely and frequently are such as to refer not only to properties of the color but also to related properties of the stimulus. Hence it seems necessary, in the interests of unambiguous thinking, to introduce certain refinements and possibly some innovations in terminology at this point. The Committee suggests the following nomenclature.

(*a*) **Brilliance[6] is that attribute of any color in respect of which it may be classed as equivalent to some member of a series of grays ranging between black and white.** Synonymous terms, as used by various writers, are "luminosity" (Abney, **4**, 4, 86) (Rood, **89**, 33) (Troland, **93**, 948), "brightness" (Luckiesh, **55**, 1) (Helmholtz, **21**, 243–245), "tint" (Titchener, **92**, 61–64), "value" (Munsell, **61**, 12–13), and "visual brightness" (Nutting, **63**, 300).

(*b*) **Hue is that attribute of certain colors in respect of which they differ characteristically from the gray of the same brilliance and which permits them to be classed as reddish, yellowish, greenish, or bluish.** There is a very satisfactory agreement among authorities regarding the usage of this term, which seems not to have been corrupted by any definite physical application.

(*c*) **Saturation is that attribute of all colors possessing a hue,**

[4] As, e.g. in the English Translation of Planck's "Theory of Heat Radiation," (74).

[5] On the definition of color as a psychological entity see: (69, 1), (73, 21–23), (94).

[6] The substitution of the word "brilliance" for the commonly used "brightness" and "luminosity" is necessitated by the fact that both of the latter terms have received technical definitions in connection with photometric measurements. It is impossible either to discard these technical definitions or to identify them with the definition here offered for the term "brilliance."

which determines their degree of difference from a gray of the same brilliance. Synonymous terms, as used by various writers, are "purity"[7] (Rood, **89,** 32; Nutting, **67,** 139; Abney, **4,** 4) and "chroma" (Munsell, **61,** 12–14; Titchener, **92,** 62–63).

(*d*) Auxiliary Terms: The term *chromaticity* may be used to characterize a color qualitatively without reference to its brilliance. Chromaticity is determined by hue and saturation together, a gray being specified by the statement that it has *no* chromaticity.[8]

(*e*) Interdependence of the Attributes: All colors except absolute black exhibit brilliance, but grays have zero saturation, and hence no hue. All colors which exhibit a hue must also exhibit saturation, and *vice versa*.

(*f*) *Species of Colors.* Colors can be classified into *chromatic* and *achromatic* species, according as they do or do not exhibit hue, respectively. The former may be designated briefly as *chromas* (including colors of all finite degrees of saturation) and the latter as *grays* (including black and white).

Median gray (="mid-gray") is the middle member of a series of grays in which each member differs from its immediate neighbors by the least perceptible difference, and of which black and white are the terminal members. This gray furnishes the most practicable reference point for the achromatic as well as for the chromatic series of colors.

Median colors are all colors equivalent in brilliance to median gray, including the latter.

Tints and shades are colors, including grays and chromas, which are respectively lighter or darker than median gray.

D. PSYCHOLOGICAL PRIMARIES.—The psychologically primary colors are those which are necessary and sufficient, in minimum number, for the description of all colors by introspective analysis.

[7] Many of the authorities mentioned fail to distinguish between the subjective attribute of color, which is designated in the present report by the term saturation, and the ratio of homogeneous to total radiation in the stimulus, or the *purity*.

[8] The term *chromaticity* as applied to a color is a natural substitute for the term *quality* which is sometimes employed to distinguish that aspect of a color which excludes its "intensity." The use of *quality* in this context is undesirable on account of the more general meaning which it possesses in psychology.

For normal vision these primaries are: black, white, red, yellow, green, and blue. (**73**, 251–252; **94**, 21). Red and yellow may be grouped together under the designation of "warm," while green and blue may be classified under the designation of "cold" primary chromas.

E. THE MEASUREMENT OF COLOR.–The three attributes of color can be treated as quantities and specified numerically, if all discriminable colors are conceived to be arranged into a system such that neighboring members differ from one another in each of the three attributes by just noticeable degrees (or threshold steps). (**92**, 207–215; **13**, 1–10). Such a system (*vide infra*) is necessarily three-dimensional (**61**, 18–31), and three ordinal values, representing the positions of a given color in the several dimensions are needed to define the color. The spectral chroma scale, considered more in detail below, is an application of this principle of color measurement to the study of the dependency of chromaticity upon wave-length.

2. STIMULUS TERMS

A. RADIANT ENERGY.–The adequate stimulus of color consists of radiant energy of certain frequencies or wave-lengths which have various stimulus values depending on the type of visual response system under consideration. The term "radiation" is often employed as a brief equivalent of "radiant energy," although this usage tends to confuse the *process* of radiation with the outcome of the process.[9]

B. THE PHYSICAL SPECTRUM is an arrangement of radiant energies in order of their respective frequencies or wave-lengths. It should not be confused with the *color spectrum* which is a series of colors aroused by part of the physical spectrum.

C. SPECTRAL DISTRIBUTIONS. The properties inherent in any sample of radiant energy which determine its capacity as a color stimulus are completely specified by its *spectral distribution*, which expresses the "intensity" for any frequency (or wave-length) as a function of the frequency (or wave-length) in question.

[9] Cf. (37).

(*a*) When plotted in the form of a curve, the *ordinates* of a spectral distribution represent "intensity" per abscissa unit (frequency or wave-length, as the case may be); and the intensity concept, for the essential case of the incidence of the radiant energy upon the retina, will be: energy per second per unit area. To be completely specific, the function must express absolute values, but this is often difficult in practice.

(*b*) The *wave-length unit* which is ordinarily employed in colorimetrics is the millimicron which is correctly symbolized by $m\mu$ (not $\mu\mu$).[10]

(*c*) It is to be noted that wave-length, strictly interpreted, does not furnish a reliable specification of the color-stimulating capacities of radiant energy, as the response of the visual system depends upon *frequency*, while wave-length may vary independently of frequency. Since wave-lengths can only be interpreted in colorimetrics as reciprocal representations of frequency, it would be desirable theoretically to employ frequency directly in formulating spectral distributions. A suggested unit of frequency is the *fresnel*, defined as one vibration per trillionth (10^{-12}) of a second. Table 1 provides means for interconverting between millimicrons and fresnels.

(*d*) Spectral distributions of *transmission*, *reflection*, *luminosity*, etc., which are often employed to specify "color," may be regarded as constituents or as developments of the essential distribution function (*vide infra*).

D. HOMOGENEOUS RADIANT ENERGY—for the purposes of colorimetrics—is radiant energy, sensibly all of the intensity of which lies within a single spectral region so small as to exhibit—under the conditions most favorable for discrimination—no perceptible hue difference within the region.

E. PURITY—The purity of any sample of radiant energy, with respect to any one of its constituents, may be defined in general as the ratio of the intensity of this constituent to the total intensity of the sample. By *physical purity* we may mean such a

[10] On the use of the symbolism $m\mu$ instead of $\mu\mu$ see C. E. Guillame, Unités et Étalons, p. 7, Paris, 1893; also Soc. Fran. de Phys., Recueil de constants physiques, p. 1; B. S. Tech. Pap. **119**, p. 7.

ratio in which the intensity is measured in energy terms, while *photometric purity* may be defined as a similar quantity based upon evaluations in terms of light units. Although the choice of the particular constituent with respect to which the purity is to be estimated is necessarily more or less arbitrary, we may define —as a special case of considerable importance—the *colorimetric purity*, which is the ratio in luminosity terms, between the dominant homogeneous constituent and the total sample, where the "dominant homogeneous constituent" comprises a range of wavelengths not greater than that corresponding to a single chromaticity threshold in the given spectral region, and has a dominant hue identical with that of the total sample, the intensity of the homogeneous constituent being arbitrarily so adjusted with respect to the total intensity that it can be mixed with "gray light" in such proportions as to yield a color-match with the total sample. This last definition corresponds with that of "per cent. white" in the method of colorimetry by "monochromatic analysis," but evidently involves psychophysical considerations in so essential a manner as to have very little physical significance.

F. MODE OF INCIDENCE—The color which is evoked by any adequate stimulus depends not only upon the spectral distribution of the latter, but also upon certain further conditions which may be called those of its *mode of incidence*. These conditions include: (1) the type of color system possessed by the observer, (2) the portion of the retinal field stimulated, (3) the size of the field, (4) the momentary state of adaptation of the optic nervous mechanism, and (5) the excitation processes in adjacent visual areas. In accurate work these factors require specification. In general, we assume pure cone vision of the normal trichromatic system, central fixation, a size not exceeding three degrees, and a gray contrast field of the same apparent brightness as the given stimulus light. (*vide infra*, for conditions of pure cone vision.)

3. PSYCHOPHYSICAL TERMS

This section deals with the terminology of the relation between color and its stimulus. The study of this relation constitutes the science of *color sensation*.

TABLE 1

Equivalents in Terms of Fresnels of Wave-lengths in Millimicrons

	Wave-Length Millimicrons	Frequency Vibrations Seconds 10^{12}		Wave-Length Millimicrons	Frequency Vibrations Seconds 10^{12}		Wave-Length Millimicrons	Frequency Vibrations Seconds 10^{12}
	400	750.0	He	492.2	609.5		600	500.0
Hg	404.7	741.3		500	600.0		610	491.8
Hg	407.8	735.7	He	501.6	598.1	Hg	615.2	487.7
	410	731.7		510	588.3		620	483.9
	420	714.3		520	576.9		630	476.2
	430	697.8		530	566.1		640	468.8
H	434.1	691.1		540	555.6		650	461.6
Hg	435.8	688.4	Hg	546.1	549.4	H	656.3	457.1
	440	681.9		550	545.4		660	454.6
He	447.2	670.8		560	535.8	He	667.8	449.3
	450	666.6		570	526.2		670	447.8
	460	652.2	Hg	576.9	520.0		680	441.2
	470	638.4		579.1	518.0		690	434.8
He	471.3	636.5		580	517.2		700	428.6
	480	625.0	He	587.6	510.5	He	706.5	424.6
H	486.2	617.1		590	508.5		710	422.6
	490	612.3						
Hg	491.6	610.2						

A. PSYCHOPHYSICAL FUNCTIONS.–There are two important, general types of psychophysical functions which occur in colorimetrics, (1) a type which expresses a direct relation of dependency between a psychological color attribute (*vide supra*)—measured in threshold steps—and a stimulus variable, and (2) a type which formulates relations between two or more stimulus variables, such relations depending upon and expressing the conditions for the *equation* in one or more psychological dimensions, of the colors due to different stimuli.[11]

B. COLOR EXCITATIONS AND PHYSIOLOGICAL PRIMARIES.–An example of the second type of function appears in the color excitation curves for various visual color systems, which curves show in what proportions of intensity a number of selected color

[11] A further type specifies the stimulus conditions for just-noticeable (or otherwise standardized) differences between the colors evoked by compared stimuli. On certain assumptions, functions of this type can be integrated to yield those of type (1).

stimuli must be mixed to match the homogeneous stimuli of the spectrum. (48; 4, 15, 223–247).

(*a*) Any set of component stimuli, thus applied, may be regarded as physiological primaries, but when they are so chosen for any or all visual color systems as to account for the maximum number of facts they may be called the *fundamental physiological primaries*. Such primaries are stimuli, not colors; although they may properly be said to "have" a color. They are of the *additive* type because the stimuli are *added* to produce the required effects.

(*b*) The corresponding *subtractive*, or "pigment," primaries are determined roughly by the spectrophotometric complementaries of the additive primaries. However, they do not consist of radiant energy but of absorbing mechanisms of one sort or another. In general they absorb from a "gray light" spectral distribution—of a certain intensity—(*vide infra*) portions which are as nearly as possible equivalent in color mixture value to the respective, additive primaries.

C. VISIBILITY, LIGHT, AND LUMINOSITY.[12] Another psychophysical function of the second type is the so-called *visibility curve*, which expresses reciprocally the intensities of radiant energy of different frequencies (or wave-lengths) which are required to match a standard in brilliance alone. The most recent average visibility values are given in Table 2.

(*a*) The conception of *light*, which is fundamental to *photometry*, is a psychophysical quantity defined as the product of the *absolute* power and visibility measures for any given sample of radiant energy.

(*b*) *Relative* light quantities are called *luminosities*.

D. COMPLEMENTARY STIMULI are stimuli which when mixed additively in certain required proportions evoke a gray.

(*a*) *Complementary colors* are the colors evoked by these stimuli, in the given proportions, separately.

E. The definition of color as a strictly psychological entity does not preclude the legitimate use of such convenient expressions

[12] For photometric concepts, see the report of the Committee on Nomenclature and Standards of the I. E. S. published each year in the Transactions of the Illuminating Engineering Society.

TABLE 2

Average Normal Visibility Values

Adopted as Standard by the Illuminating Engineering Society and the Optical Society of America 1919–1920.[13]

Wave-Lengths	Adopted Mean I. E. S	Absolute Visibility
400	0.0004	0.27
10	.0012	0.80
20	.0040	2.7
30	.0116	7.72
40	.023	15.3
450	0.038	25.3
60	.060	40.0
70	.091	60.7
80	.139	92.6
90	.208	139
500	0.323	216
10	.484	322
20	.670	446
30	.836	557
40	.942	629
550	0.993	662
60	.996	663
70	.952	641
80	.870	580
90	.757	502
600	0.631	421
10	.503	336
20	.380	253
30	.262	175
40	.170	113
650	0.103	68.8
60	.059	39.3
70	.030	20.0
80	.016	10.7
90	.0081	5.4
700	0.0041	2.7
10	.0021	1.4
20	.0010	0.67
30	.00052	0.35
40	.00025	0.17
750	0.00012	0.08
60	.00006	0.04
Wave-Length of maximum	556	

[13] Cf. JOUR. OPT. Soc. of America, 4, p. 58; 1920.

as "*the color of light*," "*the color of a material*," "*spectral colors*," etc., since such expressions may be taken to imply a psychophysical linkage between stimulus and color which is reliable under normal conditions.

III. STANDARD PSYCHOPHYSICAL DATA

The purpose of the ensuing Part of this Report is to present data on certain laws or conditions which are fundamental to visual response and, in particular, to the science of colorimetrics. These data refer mainly to psychophysical relations of the two types defined in the preceding Part, but also to certain purely psychological and purely physiological laws. A great deal remains to be discovered and made definite in this field, and the following statements merely represent the best determinations available when the best are often far from good. Reference should be had to the Report of the Committee on Visual Sensitometry for a more detailed treatment of visual laws which are of interest mainly to the photometrician or the illuminating engineer.

1. THE PSYCHOLOGICAL COLOR SOLID

The three-fold attributive nature of color permits the symbolic arrangement of all possible colors in the form of a geometrical, or quasi-geometrical solid, neighboring members being separated by just noticeable differences. At present there are not sufficient data to permit of an accurate construction, but the following approximations would appear to be determined.[14]

A. GENERAL COÖRDINATES.—Since the hues are cyclical in their resemblances, they will be represented most appropriately by angular measures with reference to a fixed point and line in any plane. The saturation must then be determined—on the general principle of polar coördinates—by a distance in the plane from the fixed point, or pole. The third dimension, of brilliance, consequently becomes a distance perpendicular to the selected plane.

B. REFERENCE AXES.—The most natural origin of coördinates is the point on the axis of the figure which represents the median gray (*vide supra*), the axis itself (standing for all of the achromatic

[14] For discussions of the Psychological Color Solid see (92, 60-64; 61, 22-25).

colors) serving as the reference line for saturation, while a plane passing through the axis and the extreme spectral red forms the reference plane for hue. (See Fig. 1.)

C. BOUNDARIES.—It is certain that the bounding surfaces of such a color solid will have an irregular contour. The axial dimen-

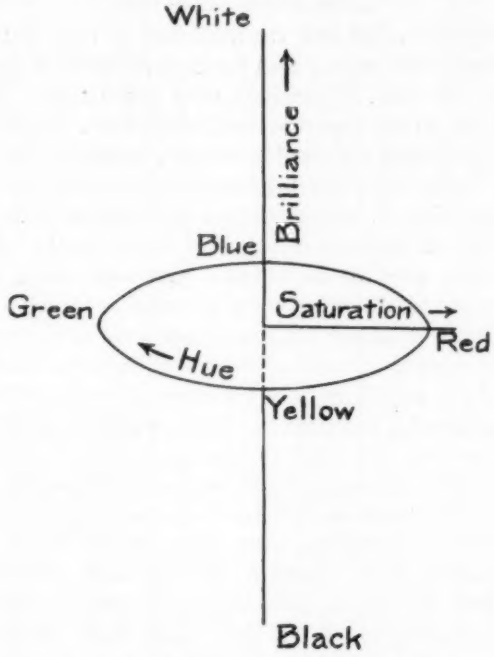

FIG. 1. The Dimensions of Psychological Color Solid

sion will be about thirty times the radial one at the greatest value of the latter. Both in the directions of high and low brilliance, there will be convergence towards a vertex. If the edges of planes perpendicular to the axis are regarded as determined by the saturations obtainable by *spectral stimuli*, these edges will be maximally near the axis at the yellow, receding on either side to find a constant value for the intermediaries of red and blue.

D. PSYCHOLOGICAL PRIMARIES.—It is not legitimate to represent the psychological primaries, as Titchener does, as corners in a quadrilateral construction, since there is no correlation between psychological primacy and saturation, which such a construction implies.

2. THE SPECTRAL CHROMA SCALE

The functions which link the attributes of color, expressed in threshold units (*vide supra*) with the characteristics of the stimulus have thus far been determined only imperfectly. Reference should be had to the Report of the Committee on Visual Sensitometry (**68**) for data on the relation between brilliance and stimulus intensity. Jones (**42**), utilizing measurements made by Steindler, Nutting and himself, has determined the function connecting the chromaticity of spectral colors with wave-length. Since the spectral colors, even for conditions of pure cone vision, are by no means of equal saturation (*vide infra*) Jones' so-called "hue scale" is in reality a resultant hue-saturation scale of a very special kind. However, as an index of the law of change of chromaticity with respect to wave-lengths of homogeneous radiation, his results are of fundamental importance. They are reproduced in Table 3. Each chromaticity unit corresponds to one just noticeable difference (both in chromaticity and in wave-length), and the reciprocal of the wave-length difference is the sensibility to change in wave-length. It will be noted that this sensibility has four distinct maxima in the spectrum, the two most important ones lying at 494 and 588 $m\mu$, respectively, where the wave-length threshold is approximately 1.0 $m\mu$. Jones finds 128 just noticeable chromaticity steps in the spectrum, and about 20 additional steps in the non-spectral purples and magentas, as determined by their complementaries.

Since the hues form a cyclic series, it would seem more appropriate to express the hue scale in angular than in linear notation. If there are H hue steps in the complete cycle, the angular unit will evidently be $2\pi/H$ radians. The magnitude of this unit, however—if it is to correspond always to an integral step—must vary with the saturation, so that the linear unit is probably the more convenient. No determinations have yet been made of the

TABLE 3
Spectral Chroma Scale

No.	λ(mμ)	dλ	No.	λ	dλ	No.	λ	dλ
1	700.0	.....	44	576.5	1.4	87	490.4	1.1
2	678.0	22.0	45	75.2	1.3	88	89.4	1.0
3	65.0	13.0	46	73.7	1.5	89	88.2	1.2
4	59.0	6.0	47	71.7	2.0	90	87.0	1.2
5	54.0	5.0	48	70.1	1.8	91	85.8	1.2
6	49.5	4.5	49	68.4	1.7	92	84.5	1.3
7	46.0	3.5	50	66.6	1.8	93	83.2	1.3
8	42.8	3.2	51	64.8	1.8	94	81.7	1.5
9	40.2	2.6	52	63.0	1.8	95	80.0	1.7
10	37.8	2.4	53	61.1	1.9	96	78.2	1.8
11	35.5	2.3	54	58.6	2.5	97	76.5	1.7
12	33.1	2.4	55	57.0	2.6	98	75.0	1.5
13	30.0	3.1	56	54.4	2.6	99	72.9	2.1
14	26.5	3.5	57	51.8	1.6	100	70.5	2.4
15	23.0	3.5	58	49.1	2.7	101	68.2	2.3
16	20.0	3.0	59	46.1	3.0	102	65.8	2.4
17	17.3	2.7	60	43.0	3.1	103	63.6	2.2
18	14.9	2.4	61	.8	3.2	104	61.2	2.4
19	12.5	2.4	62	36.5	3.4	105	58.7	2.5
20	10.2	2.3	63	33.2	3.3	106	56.5	2.2
21	08.0	2.2	64	30.1	3.1	107	54.4	2.3
22	06.0	2.0	65	27.1	3.0	108	52.1	2.3
23	04.1	1.9	66	24.2	2.9	109	50.0	2.1
24	02.3	1.8	67	21.4	2.8	110	48.0	2.0
25	600.6	1.6	68	19.1	2.3	111	46.0	2.0
26	599.0	1.6	69	16.8	2.3	112	44.2	1.8
27	97.4	1.6	70	14.6	2.2	113	42.5	1.7
28	95.9	1.5	71	12.6	2.0	114	40.8	1.7
29	94.5	1.4	72	10.6	2.0	115	39.0	1.8
30	93.1	1.4	73	08.0	1.6	116	37.2	1.8
31	91.8	1.3	74	07.0	1.0	117	35.3	1.9
32	90.5	1.3	75	05.4	1.6	118	33.3	2.0
33	89.5	1.0	76	04.0	1.4	119	31.3	2.0
34	88.5	1.0	77	02.6	1.4	120	29.3	2.1
35	87.5	1.0	78	01.3	1.3	121	27.0	2.2
36	86.4	1.1	79	500.0	1.3	122	24.8	3.2
37	85.3	1.1	80	498.7	1.3	123	22.3	2.3
38	84.0	1.3	81	97.4	1.3	124	19.7	2.8
39	82.7	1.3	82	96.1	1.3	125	16.7	3.0
40	81.5	1.2	83	94.8	1.3	126	13.8	2.9
41	80.3	1.2	84	93.7	1.1	127	10.4	3.4
42	79.1	1.2	85	92.6	1.1	128	405.8	4.6
43	77.9	1.2	86	91.5	1.1	.....	.....	.....

number of hue steps in cycles of color with uniform saturation, and it is quite uncertain whether the change in number of steps follows the simple geometrical analogy: $H = 2\pi s$, where s is the saturation measure in threshold steps from the equivalent gray. (This clearly raises the question as to whether the space of the psychological color solid is Euclidean.) The angular magnitudes in this system would preferably be measured from an axis through the normal hue of extreme spectral red, on account of the stability of this color in relation to its stimuli, and all functions of these magnitudes will be periodic.

The positions of the psychologically primary hues in the "hue scale" are matters of considerable interest. On Jones' spectral chroma scale, taking the zero in the violet and unity at extreme spectral red, the primary blue, green and yellow lie at .24, .41, and .68, respectively, the red having a value slightly in excess of 1.00, on account of the necessity of including a slight amount of blue in the stimulus for red. (**95**). It will be seen that the separations of the primaries on this scale are by no means equal.

3. THE SATURATION SCALE

Careful determinations of the number of just noticeable saturation steps between each maximally saturated color and white have not yet been made. Nutting and Jones (**67**; **68**) find about 20 such steps for red, green and blue, the thresholds for change in the per cent. white varying with the given per cent. white as shown in Table 4.

TABLE 4

Saturation Scale Data

Per cent. white	0	10	20	30	40	50	60	70	80	90	100
Threshold white	4.7	4.6	4.5	4.4	4.2	4.0	3.7	3.4	3.0	2.5	2.1

These values, like those for wave-length sensibility, are independent of brilliance over a wide range of intensities.

The Chairman has attempted to determine the relative saturations of the spectral colors by an application of the flicker photometer. It is assumed that when any color is alternated with

a white of the same brilliance the critical flicker frequency is a function of saturation alone, so that colors possessing the same critical frequency will be of equal saturations. Since the spectral colors have radically different frequencies, with a minimum in the yellow at 575 $m\mu$, it is necessary to add white to all except the latter to attain this equality. The per cents. of white (in terms of the total mixture) which were found necessary for the writer's right eye are shown in Table 5.

TABLE 5
Comparative Saturations of Spectral Colors

Wave-length, $m\mu$.....	419	438	458	479	497	517	537	556
Per cent. white.....	53.0	75.1	63.5	51.2	49.0	30.0	28.9	19.0
Wave-length, $m\mu$.....	575	595	614	634	653	673	692	.....
Per cent. white.....	0.0	19.3	38.8	31.5	46.7	50.7	49.4	.....

These figures are only tentative, as the sources of error in the experiment seem quite numerous. However, they indicate quite clearly that the differences in saturation of the spectral colors are of first order importance, so that it is highly improbable that the number of saturation steps from white is the same for them all. These conclusions appear to be corroborated by the relations of the spectral colors to white as represented on the color-mixture triangle (*vide infra*).

4. STIMULI FOR THE PSYCHOLOGICAL PRIMARIES

Westphal (**98**) has determined stimuli, as nearly as possible homogeneous, for arousing the psychological primaries in the average normal observer. They are: for red, extreme visible long-wave end of the spectrum plus a small amount of blue or violet; for yellow, 574.5 $m\mu$; for green, 505.5 $m\mu$; and for blue, 478.5 $m\mu$. It is interesting to note that three of these stimuli (for red, green and blue) correspond quite closely with the three fundamental physiological primaries determined by König and Dieterici.

5. THE COLOR EXCITATION FUNCTIONS

Probably the most fundamental of all the psychophysical data relating to color are the *three-color excitation curves*, which represent

the laws of three-color mixture. Extant data on these relationships are due to Maxwell (**57**), Abney (**4**), and König and Dieterici (**48**), the results obtained by the first investigator, however, being at present of relatively little value. The remaining two investigations were made with quite different light sources and choices of reference points, but are capable of being reduced to a common denominator by appropriate calculations. This work has been carried out very painstakingly by Mr. E. A. Weaver of the present Committee, who finds that the two sets of data actually agree surprisingly closely, so that they may legitimately be averaged. The results, reduced to an equal energy spectrum and referred to average noon sunlight as an origin (in trilinear coördinates) are given in Table 6 and Fig. 2.

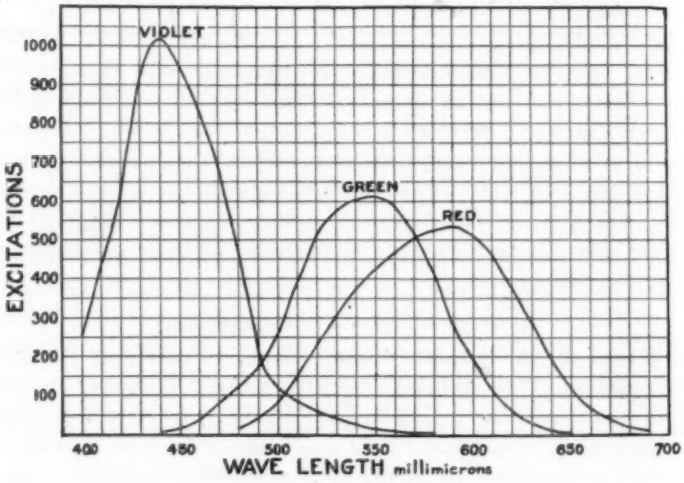

FIG. 2. Elementary Color Excitations for Different Wave Lengths
(The ordinates correspond with the values listed under "Excitations" in Table 6.)

A detailed account of the method employed in the reduction of the two sets of data will be given in a separate publication, but may be outlined briefly here as follows. Abney's luminosity curve values were first converted so as to refer to average noon sunlight

instead of to the carbon arc (as given), using Watson's data[15] for the energy distribution of the latter and Abbot's (1) data on that of the former (*vide infra*). The corresponding luminosity and percentage color excitation values were then combined as products to yield excitation values referred to sunlight, rendering them com-

TABLE 6

Spectral Colors in Terms of Elementary Excitations

Wave-Length mμ	Excitations			Percentages		Wave-Length	Excitations			Percentages	
	Red	Green	Violet	Red	Violet		Red	Green	Violet	Red	Violet
400	.....	.....	253	.....	100	550	424	612	18	40.2	1.7
410	.....	.....	433	.....	100	560	466	578	11	44.2	1.0
420	.....	.....	614	.....	100	570	505	517	7	49.5	.7
430	.....	.....	915	.....	100	580	520	415	4	55.4	.4
440	.....	7	1019	.....	99.3	590	535	296	...	64.3	.....
450	.....	16	950	.....	98.3	600	510	196	...	72.2	.....
460	.....	38	842	.....	95.7	610	462	113	...	80.4	.....
470	.....	81	697	.....	89.6	620	375	59	...	86.3	.....
480	14	122	473	2.3	77.6	630	285	29	...	90.8	.....
490	41	169	220	9.5	51.1	640	195	10	...	95.1	.....
500	83	260	123	17.8	26.4	650	118	3	...	97.5	.....
510	151	391	87	24.0	13.8	660	68	.....	...	100	.....
520	233	510	61	29.0	7.6	670	40	.....	...	100	.....
530	307	572	43	33.3	4.7	680	22	.....	...	100	.....
540	373	603	29	37.1	2.9	690–750	27	.....	...	100	.....

These values are for an equal energy spectrum. The relative magnitudes of the three elementary excitations have been chosen so that the curves for average noon sunlight have equal areas; that is, if the percentage values are plotted on a trilinear diagram, sunlight falls in the center. The absolute excitation values are based upon a convenient arbitrary unit. The percentage values are given for the red and violet only since those for the green can be found by subtracting the sum of the other two values from 100 in each case.

parable in this respect with the values given by König and Dieterici. The respective trilinear representations of both sets of values then coincided, except for the positions of the elementary "green" excitations and the sides of the two triangles joining the green and the "violet" elementaries. Although the relation between the two green elementaries could not be determined directly, it

[15] See (5, p. 96, Table 1 and p. 97, Fig. 3).

was established indirectly by use of the known positions of the solar white in each of the triangles, this correlation permitting the reduction of the two sets of values to three common elementaries. The red and green values of each set were now tested separately against the data of independent color matches published by Priest (**79; 81**) in connection with his investigations of the leucoscope and camouflage paints, which accidentally provided materials for checking the results. Both sets checked equally well, so that they were given equal weight in the computation of average values. The violets were tested by data on complementaries and that of Abney was rejected. These best values were then retransformed to terms of elementaries determined by a triangle based on extreme spectral red and violet, with its sides as closely tangent to the locus of the spectral colors as possible. Finally the excitation values were reduced to terms of an equal energy spectrum. Thus expressed, the areas under the three curves are equal for the energy distribution of average noon sunlight, the magnitudes or "weights" of the three elementaries having been so chosen as to yield the solar white with equal excitations.

When quite differently weighted, in terms of the relative powers of the three elementary processes to generate brilliance, the three chromatic curves should summate to yield the visibility curve. It is a well-known fact that in this summation the value of the violet or blue excitation is extremely small compared with that of the red and green. König and Dieterici give no data from which these specific visibility coefficients of the chromatic processes can be deduced. Abney, however, provides data[16] of this sort leading to coefficients by which the ordinates of the excitation curves must be multiplied in order that all three curves should summate to yield his own visibility curve. This latter curve, however, as derived from Abney's luminosity curve and carbon arc data, departs so widely from the average visibility function, as specified by the Standards Committee of the Illuminating Engineering Society, as to throw doubt upon the general validity of these values. Mr. Weaver and the writer have made

[16] See (**4**, Table 38, p. 239, and Table 34, p. 17, Columns 7 to 9). Also (**17**).

new experimental determinations of the chromatic visibility coefficients for their own eyes and those of one other subject. The results not only differ from Abney's, but show such large variations among themselves as to indicate that these coefficients are subject to marked fluctuation among individuals. All three of our subjects agree perfectly on the proportions of the mixed stimuli required for the color matches (measured in energy units), but disagree on the photometry of the components. In other words, their visibility curves vary widely while their color curves remain constant. This may indicate either that the brilliance process is independent of the chromatic processes or that the latter, as constituents of the former, vary in weight without alteration in the mathematical form of their response functions. The average values for the factors obtained from our three subjects were: red, 0.370; green, 0.617; and blue, 0.012.

The stimulation values of all forms of radiant energy, simple or complex, can be completely specified in terms of the ratios of excitation of the three chromatic elementaries,—combined with one absolute measure of intensity, if the brilliance as well as the chromaticity of the color is to be taken into consideration. To arrive at the expression of the chromatic stimulation value of any given distribution of radiant energy in terms of the three elementaries, it is only necessary to multiply each of the values for the elementaries given in Table 6 by the corresponding ordinates of the distribution function throughout the spectrum and then to find the areas under the resulting three curves. The ratio of these areas determines the chromaticity of the color. The reduction of data in one system of color specification to data of another system (*vide infra*) can be accomplished with the greatest sureness of principle through the medium of a common expression in terms of the three elementaries. For the science of color the three elementaries are far more fundamental even than the spectrum, and the expression of the spectral colors in terms of these components (as in Table 6) is as natural as (say) the similar expression of various Planckian distributions (see Table 17).

It is, of course, improbable that the curves in Fig. 2 faithfully represent the actual resonance functions of the elementary chro-

matic mechanisms in the retina. There is also little doubt, however, that the curves in question and all specifications based upon them, are potentially convertible into terms of such actual response functions. The characteristic constants of these latter functions must be determined from data auxiliary to the facts

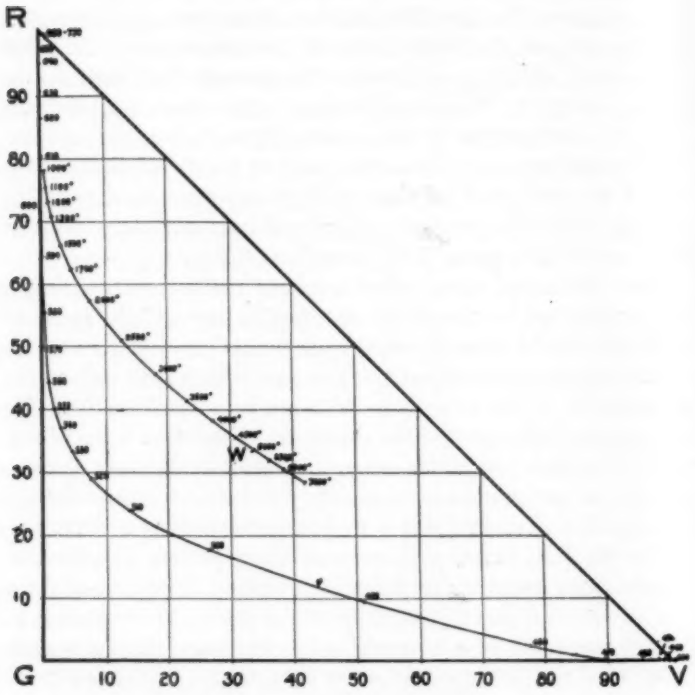

FIG. 3. Color Triangle, Showing Loci of Spectral and Black Body Excitations

(The ordinates give the percentage excitation values for Red and the abscissae those for Violet.)

of color-mixture for the normal eye, such as those of color-blindness and chromatic minuthesis. Attempts to arrive at so-called *fundamental excitations*, based upon these more comprehensive considerations, have been made by König, Abney, Exner (17) and others.

but the results cannot be regarded as sufficiently final to justify their adoption in place of a maximally straightforward representation of the facts of color-mixture, such as is given in Table 6.

Fig. 3 provides a graphical representation of some of the relations based upon Mr. Weaver's analysis, in terms of trilinear coördinates or a so-called color-mixture triangle. It will be noted that the triangle here given is rectangular rather than equilateral, as is ordinarily the case. The latter form bears the simplest relation to the representation of the three color excitation values in the familiar three-dimensional Cartesian coördinates, being a

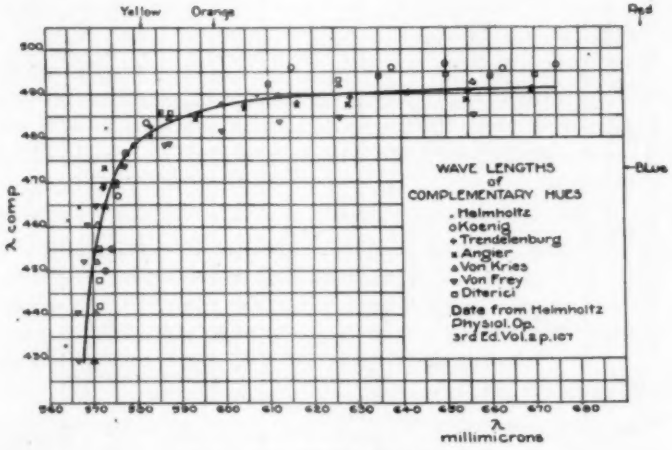

FIG. 4a. Wave-Lengths of Complimentary Hues

section of this system making equal angles with all of the reference planes. The rectangular form, however, is much easier to use either in plotting the loci of colors or in determining the excitation values of the colors lying on curves already plotted. The result of the mixture of stimuli represented by two or more points in the triangle is found by computing the position of the "center of gravity" or centroid, of the multiple-point system, in which the intensities of the components—expressed in relative units defined by the triangle—function as analogues of masses.

6. COMPLEMENTARY COLORS AND WAVE-LENGTHS

Among the most interesting results of the mixture of different stimuli are the identification of pairs of homogeneous radiations which, with the proper ratio of intensities, yield white. Such *complementary wave-lengths* can be found in the color-mixture triangle at the two intersections of straight lines, drawn through the central white, with the locus of the spectral colors. It is

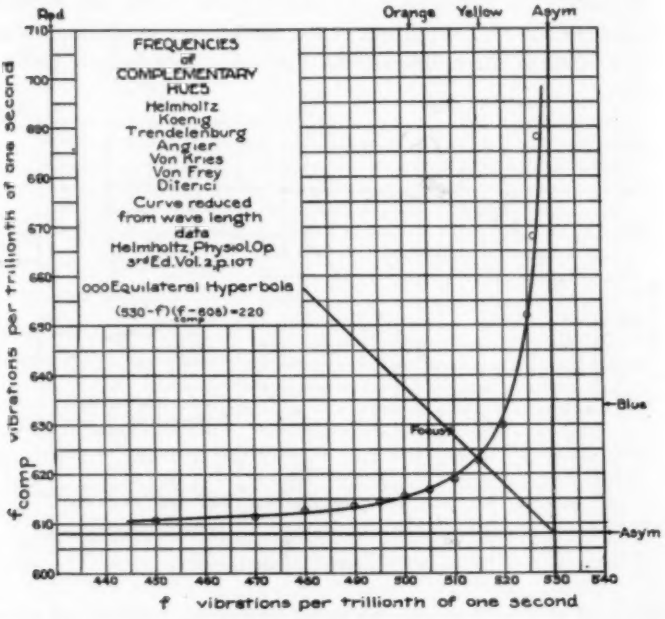

FIG. 4b. Frequencies of Complementary Hues

clear from an examination of the triangle that there is a range of spectral colors—in the green—which have no single spectral complementary, the complementaries of either end region of the spectrum being found in the other. Extant data on complementary wave-lengths are unsatisfactory because of indefiniteness in the specification of the white stimulus which was employed. Fig. 4a gives a plot of the best values which are available. In

Fig. 4b the plot of these values in terms of frequency shows how closely their relation fits the function expressing a rectangular hyperbola, having an equation:

$$(530-f)\ (f_c-608)=220,$$

f being the given frequency and f_c its complementary. (**80**)

7. STANDARD CONDITIONS FOR PURE CONE VISION

In order to secure reliable conditions for complete color vision even in the normal observer, it is necessary to restrict the stimulus to the retinal cones, excluding the rods, which yield only achromatic colors. Pure cone vision can be secured by satisfying the following requirements.

A. CHOICE OF OBSERVERS.—Recent investigations by Abney indicate that a considerable number of individuals possess rods in the center of the retina as well as in the periphery, so that before relying upon the restriction of the stimulus to a central field, the observer should be tested for the Purkinje phenomenon in central vision.

B. SIZE OF FIELD.—The normal retina possesses no rods in an area slightly greater than three degrees in diameter, surrounding the intersection of the line of sight with the retina (**73**, 10). Consequently, in the case of an observer known to be normal in this respect, a field of three degrees, with fixation on the center of the field, insures pure cone vision at all intensities.

C. INTENSITY.—With all observers and all field sizes, pure cone vision is obtainable at intensities above approximately one hundred photons, provided the eye has not previously been exposed for a considerable time to a much lower intensity or assuming a condition of equilibrium adaptation to the given intensity level, which should be reached within ten minutes (**75**; **34**). One hundred photons represents an external stimulus surface brightness of one hundred candles per square meter, used with a pupillary opening of one square millimeter, or equivalent conditions as regards retinal illumination.

IV. PHYSICAL STANDARDS

It is the function of the present Part of this Report to consider some physical standards which are of importance in colorimetrics.

These standards consist in certain typical forms of stimuli to color, or in factors or functions contributory to such stimuli. Some of the standards considered below are primarily of theoretical or research interest, while others are essentially of technical significance only.

1. THE CRITERION OF HOMOGENEOUS RADIANT ENERGY

As defined in a preceding Section, the criterion of homogeneity in a stimulus, for the purposes of colorimetrics, must rest upon wave-length sensibility, and hence upon the facts which are summarized in Table 3. In general, in order to be considered homogeneous, a given sample of radiation must have a range of wave-lengths not greater than the threshold for wave-length in the given region (defined by its mid-wave-length). As seen from the Table, this varies widely for different parts of the spectrum, e.g., being 22 mμ in the extreme red (680 mμ), and 1.0 mμ for 588 mμ.

2. STANDARDS OF SPECTRAL ENERGY DISTRIBUTION

Under this caption are included curves and constants indicating the distributions of "intensity" (*vide supra*) in the physical spectrum of certain frequently encountered or critically important forms of *heterogeneous* radiant energy. These distributions are all at least approximately of the so-called "black body," or Planckian type, i.e., they are determined by a general equation of the form,

$$E = \frac{c_1}{\lambda^5\left(e^{\frac{c_2}{\lambda T}} - 1\right)}$$

where E is the energy per unit wave-length, T the absolute temperature of the source, e the base of the natural system of logarithms, λ the wave-length, and c_1 and c_2 are constants. When our concern is only with chromaticity, we need consider merely *relative* energies, and any convenient value may be adopted for c_1, such as a value which makes the maximum of the function equal to unity. The value of c_2 at present recommended is 14350 micron-degrees (**19**). This equation has been found to express very closely the energy distributions for the radiation

emitted by incandescent solids, such as those contained in natural and artificial illuminants, although in the majority of cases T is not the actual temperature of the material, but is a temperature determined by the distribution itself and known as the *color temperature*, this being the actual temperature of a theoretical black body which would yield that same relative distribution in the visible spectrum. Table 7 and Fig. 5 give the relative intensities for various representative wave-lengths for Planckian distributions at a considerable variety of temperatures.

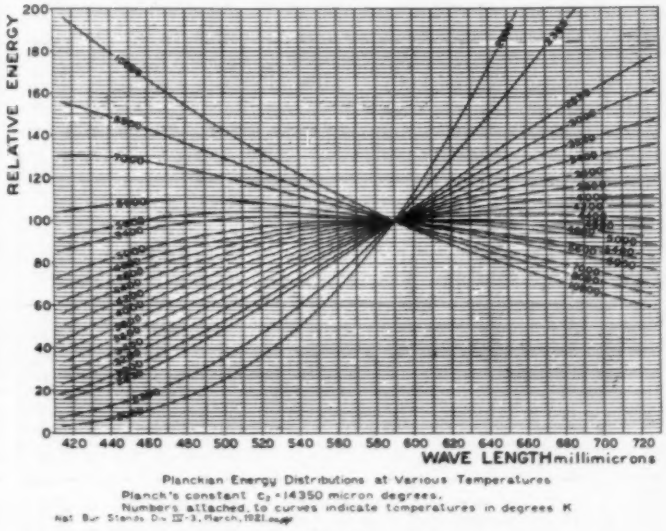

FIG. 5

A. AVERAGE NOON SUNLIGHT.—The most important standard of energy distribution, from the point of view of colorimetrics, is that which characterizes "daylight," since it is with respect to deviations from this distribution that the chromatic processes of vision have been adjusted by nature. Unfortunately, however, the form of this distribution is highly variable. There is, in the first place, the radical difference between sky-light and direct sunlight, the former exhibiting that marked deficiency in long-

TABLE 7

Relative Energy of a Black Body at Various Temperatures and Wave-Lengths

Absolute Temperature	1000	1200	1400	1600	1800	2000	2200	2400	2600	3000	3600	5000	6000
Wave-length mμ	x 10^{-15}	x 10^{-13}	x 10^{-12}	x 10^{-11}	x 10^{-10}	x 10^{-9}	x 10^{-8}	x 10^{-8}	x 10^{-7}	x 10^{-7}	x 10^{-6}	x 10^{-5}	x 10^{-5}
400	26	101	727	1787	2158	1584	809	3148	994	6256	4612	7465	7482
410	54	186	1200	2728	3101	2168	1064	4005	1230	7403	5056	7872	7878
420	111	330	1926	4037	4368	2915	1378	5026	1502	8663	5782	8244	8252
430	214	568	3024	5949	6037	3856	1757	6220	1813	10038	6407	8591	8601
440	416	954	4642	8519	8204	5022	2212	7608	2163	11520	7051	8911	8923
450	766	1559	6946	11976	10965	6449	2747	9194	2555	13111	7724	9208	9222
460	1375	2490	10210	16550	14440	8171	3373	10995	2989	14798	8373	9475	9492
470	2396	3388	14740	22510	18750	10226	4096	13020	3464	16580	9041	9716	9737
480	4091	5947	20890	30140	24030	12650	4923	15280	3982	18450	9712	9933	9957
490	6776	8933	29130	39810	30420	15480	5860	17770	4544	19975	10378	10123	10151
500	10990	13169	40030	51890	38080	18760	6914	20500	5143	24410	11039	10286	10319
510	17460	19020	54170	66820	47140	22510	8087	23480	5784	24480	11688	10429	10466
520	27230	27090	72340	85020	57780	26770	9382	26690	6464	26614	12329	10544	10586
530	41740	38000	95450	107060	70160	31580	10810	30150	7180	28780	12954	10639	10686
540	62670	52550	124300	133400	84410	36950	12370	33830	7930	30980	13563	10712	10764
550	92730	71710	160200	164600	100720	42920	14050	37750	8712	33205	14148	10762	10820
560	134900	96660	203300	201200	119250	49520	15870	41900	9524	35444	14715	10797	10862
570	193500	128700	257700	243900	140060	56740	17820	46240	10364	37675	15260	10813	10882
580	274100	169400	322140	293400	163470	64640	19900	50790	11234	39920	15782	10811	10887
590	382600	220500	399000	349900	189430	73180	22110	55530	12112	42058	16280	10793	10877

TABLE 7—*Continued*

TABLE 7—*Continued*

Absolute Temperature	1000	1200	1400	1600	1800	2000	2200	2400	2600	3000	3600	5000	5000
Wavelength $m\mu$	x 10^{-14}	x 10^{-13}	x 10^{-12}	x 10^{-11}	x 10^{-10}	x 10^{-9}	x 10^{-8}	x 10^{-8}	x 10^{-7}	x 10^{-7}	x 10^{-6}	x 10^{-6}	x 10^{-5}
600	527500	284200	490100	414600	218200	82400	24430	60460	13012	44356	16752	10763	10853
610	718400	362700	596700	487600	249800	92300	26880	65530	13936	46542	17197	10716	10815
620	968400	458800	721500	569900	284300	102850	29450	70770	14868	48702	17618	10660	10767
630	1291500	575300	866000	661800	321800	114100	32120	76120	15808	50804	18010	10590	10702
640	1704100	715300	1032100	764000	362500	126000	34910	81620	16742	52880	18332	10510	10632
650	2226100	882400	1221300	877200	406400	138500	37790	87200	17691	54900	18713	10420	10547
660	2882000	1080200	1437500	1001600	453300	151700	40760	92850	18642	56990	19024	10323	10457
670	3697900	1312900	1681400	1138200	503600	165500	43820	98610	19590	58770	19313	10218	10362
680	4706000	1585900	1955500	1287000	557100	179900	46960	104410	20540	60640	19573	10101	10252
690	5939800	1900800	2261700	1448000	613800	194900	50160	110290	21480	62400	19809	9985	10141
700	7440500	2266700	2601900	1623000	673600	210400	53420	116140	22400	64090	20022	9861	10024
710	9251200	2686000	2979200	1810000	736700	226500	56740	121980	23210	65740	20207	9732	9903
720	11419000	3165000	3395100	2012000	803000	243000	60120	127860	24220	67320	20368	9600	9776
730	14005000	3706800	3851300	2227000	872000	260000	63520	133720	25120	68820	20511	9462	9652
740	17063000	4321700	4348600	2456000	944300	277300	66960	139570	25980	70230	20627	9323	9514
750	20663000	5013000	4891000	2700000	1019500	295100	70420	145350	26840	71600	20725	9179	9377
760	24874000	5785600	5480900	2957000	1097100	313300	73910	151100	27670	72870	20828	9035	9239

All of the values in this table were calculated by means of Wien's equation with the exception of the last column, headed "5000x10-5" and that was calculated by means of Planck's equation. The values show the correct relationships for different wave-lengths at a single temperature, and between different temperatures if multiplied by the factors which are placed at the heads of the several columns. See Forsythe, Ref. 19, pp. 330–331.

wave radiations compared with the latter, which is responsible for the blue color of the sky. Direct sunlight, moreover, varies in the form of its distribution curve with the time of the day and year, and with latitude on the earth's surface. Different again, is the distribution which accompanies an overcast sky. (**54**, 37–39; **81**).

It is necessary therefore to adopt as a standard an average curve, representing the conditions most frequently encountered.

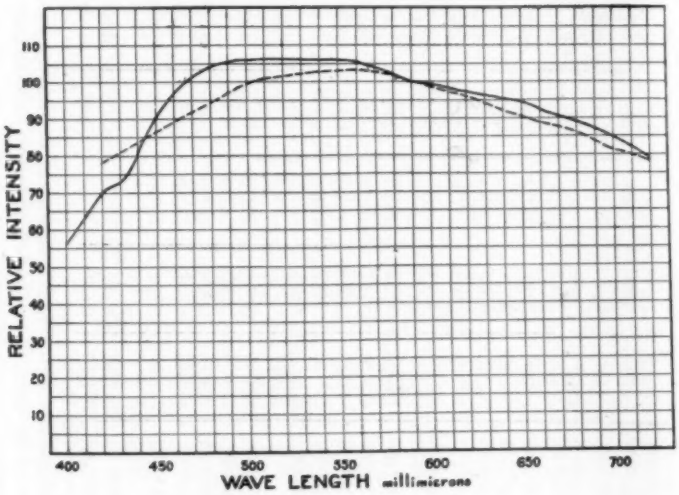

FIG. 6. Spectral Energy Distributions of Natural and Artificial Sunlight

(The solid line represents the distribution of average noon sunlight, while the broken line is that of Priest's precision artificial sunlight.)

Such an average, for noon sunlight at Washington, D. C., is given in Table 8 and the solid line in Fig. 6. It is the mean of forty determinations, half of which were made at the summer solstice (June 21) and the other half at the winter solstice (December 21), both high and low atmospheric transmissions being included. The authority is Abbot of the Smithsonian Astrophysical Observatory (**1**). Average noon sunlight, thus defined, corresponds roughly to a black body temperature of 5000°K. the distribution not being strictly Planckian.

B. STANDARD ARTIFICIAL SUNLIGHT.—There are various methods for producing artificially an approximate reproduction of average noon sunlight, as above specified. The most accurate is that of Priest which consists in passing the radiation from a gas-filled tungsten lamp, operated at a color temperature of 2848°K, (about 15.6 l. p. w., for concentrated filament lamp), through a pair of crossed nicol prisms between which is inserted

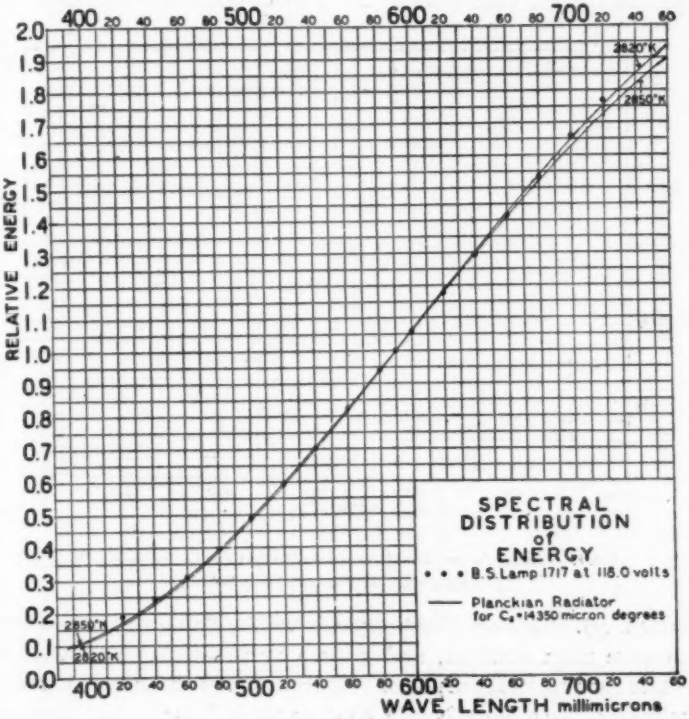
RELATIVE ENERGY
WAVE LENGTH millimicrons
SPECTRAL DISTRIBUTION of ENERGY
• • • B.S. Lamp 1717 at 118.0 volts
—— Planckian Radiator for C₂ = 14350 micron degrees
2820°K
2850°K

FIG. 7

a crystalline quartz plate 0.500 mm. thick with surfaces perpendicular to the optic axis of the crystal (78). The resulting energy distribution is shown, in comparison with that of actual sunlight, by the broken line in Fig. 6. The proper energy distribution for the radiation from the lamp, before passing through the nicol prisms and the quartz plate, is given in Fig. 7. Lamps yielding

this distribution at a specified voltage can be obtained from the Bureau of Standards, Washington.

Other methods of producing artificial daylight involve the use of blue glasses or gelatine filters before standard illuminants. The most available system of this sort at the present time consists of a No. 78 Wratten photometric filter (97) manufactured by the Eastman Kodak Company and a cylindrical acetylene flame (41) produced by a standard burner (also obtainable from the same

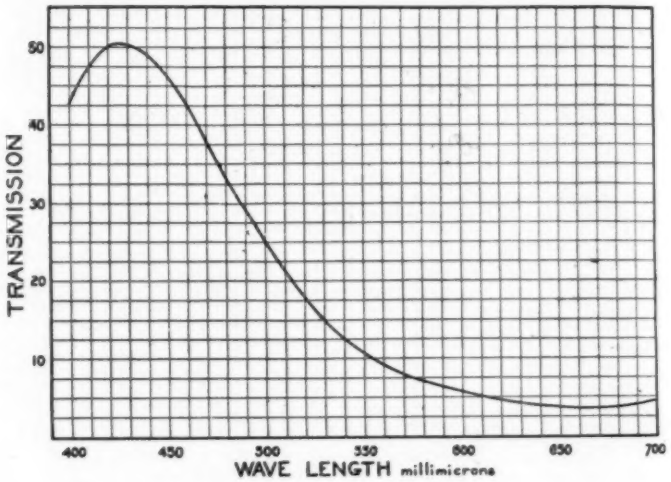

FIG. 8. Spectral Transmission of Tungsten-to-Daylight Filter (Wratten No. 78) (According to the Wratten Light Filter Booklet.)

company). This combination yields a white closely approximating average noon sunlight, and gives a very satisfactory standard white for practical purposes. Figure 8 shows the spectral transmission of the original No. 78 Wratten filter. It is planned to make the "78" and "86" series of filters in the future a quite accurate means of converting one color temperature to others, the original filters having been only approximate means to this end. The color temperature of the standard acetylene flame is 2360 degrees K., which corresponds with that of a vacuum tungsten filament burned

TABLE 8

Relative Intensity Values over the Visible Spectrum for Average Noon Sunlight (Computed from Abbot's data)

Wave-Length mμ	Relative Intensities		
	June 21	December 21	Mean
400	63.0	48.0	56.0
10	71.0	54.0	63.0
20	79.0	60.0	70.0
30	82.0	64.0	73.0
40	90.0	72.5	81.0
50	98.5	82.5	90.5
60	104.5	89.0	97.0
70	107.5	95.0	101.0
80	109.0	98.5	104.0
90	110.0	101.0	105.5
500	110.5	102.0	106.0
10	110.0	103.0	106.5
20	109.0	103.5	106.0
30	108.5	104.0	106.0
40	108.0	104.5	106.0
50	107.0	105.0	106.0
60	106.0	104.5	105.0
70	104.5	103.0	104.0
80	102.5	102.0	102.0
90	100.0	100.0	100.0
600	98.5	100.0	99.0
10	97.0	99.5	98.0
20	95.5	99.0	97.0
30	94.0	98.5	96.0
40	92.5	98.0	95.0
50	90.3	97.5	94.0
60	88.0	96.5	92.0
70	86.0	95.0	90.5
80	84.0	93.0	88.5
90	81.5	92.0	87.0
700	79.0	90.0	84.5
10	76.5	87.0	82.0
20	74.5	84.0	79.0

at an efficiency of approximately 8 lumens per watt. This corresponds fairly closely with the color of an ordinary mazda B tungsten light burning at 1.25 watts per mean horizontal candle power. Other selectively absorbing glasses designed to make possible the production of artificial daylight include the Ives-Brady glass, a very satisfactory but rare product, the Luckiesh "Trutint" glass, and the Corning "Daylite" glass.

C. NORMAL GRAY LIGHT.—The conception of "white light" is one which is of fundamental importance to many of the purposes of colorimetrics, for example in colorimetry by the "monochromatic" method, in defining complementaries, etc. There are a vast number of characteristic intensity distributions of radiant energy which can be used with practical success to meet this need. Although the one most frequently employed is that of average noon sunlight, to be of the greatest theoretical as well as practical significance, the definition of "white light" should evidently determine a spectral distribution which will generate a pure gray by its action on the normal human visual apparatus in a normal condition. Since there are an infinity of conceivable distributions which would satisfy this requirement, it seems advisable to limit the general form of the distribution to a species of which only one member can excite a gray. Distributions of the Planckian type meet this requirement (**32**, 198–202), and are further to be recommended because of the approximate conformity of all natural and artificial radiant sources to the Planckian law, and the comparative ease with which distributions of this sort can be reproduced in the laboratory. Preliminary determinations by Priest, (**82**), using a system of nicol prisms and quartz plates as a filter to yield Planckian distributions representative of temperatures lying between 4200 and 6200°K, indicate 5200° as the value for the gray stimulus. This figure is regarded at present as highly tentative, on account of the small number of subjects tested and doubt as to the normality of certain of them. Criteria for the selection of the pure gray, other than that of the simple introspective judgments used by Priest, may also be advisable.

TABLE 9

Color Temperatures of Common Illuminants[a]

Hefner amyl acetate lamp	1875
Pentane (10 c. p. standard)	1914
Candle (paraffin)	1920
Candle (sperm)	1925
Kerosene lamp (round wick)	1915
Kerosene lamp (flat wick)	2045
Acetylene, ordinary (as a whole)	2368
Acetylene, ordinary (central spot)	2448
Acetylene (Eastman Standard)	2360
4 W. P. M. H. C. Carbon (4.85 W. P. M. S. C.)	2070
3.1 W.P.M.H.C. Treated Carbon (3.73 W.P.M.S.C)	2153
2.5 W.P.M.H.C. Gem	2183
2 W.P.M.H.C. Osmium	2176
2 W.P.M.H.C. Tantalum	2249
1.25 W.P.M.H.C. Tungsten (Mazda B)	2385
.9 W.P.M.H.C. Tungsten	2543
Carbon arc (solid carbon)	3780
Carbon arc (cored carbon)	3420

[a] Hyde, E. P. and Forsythe, W. E., *Jour. Franklin Inst.*, **183**, p. 354; Forsythe, W. E., *Phys. Rev.* (2), **17**, p. 147, 1921; Priest, I.G., Color Temperature, Op. Soc. Am., Conv., Rochester, Oct., 1921, *J. Op. Soc. Am.*, Jan., 1922.

D. STANDARD ILLUMINANTS.—All common illuminants having a Planckian distribution are of course characterized by a temperature lower than that of the sun and of the black body which emits normal gray light, and hence evoke an unsaturated yellowish or orange color. The spectral distributions of these various illuminants can obviously be specified by a statement of their respective color temperatures. Table 9 lists these temperatures for a group of familiar light sources (**28**). The radiation from Welsbach gas mantles cannot be matched satisfactorily with that from a black body at any temperature, and varies quite widely in distribution with the proportions of ceria and thoria in the mantle as well as with the average degree of incandescence. Probably the illuminant whose characteristics are best established at the present time is the acetylene flame produced by a standard burner under specified conditions. The spectral distributions for vacuum tungsten electric lamps are determined by the efficiency, or lumens per watt, at which they are operated, and extensive measurements made at the Nela Research Laboratory (**27**), enable one to trans-

late an efficiency value into a corresponding color temperature and hence to ascertain the distribution. Table 10 shows the relation between efficiency and color temperature. Gas filled and carbon filament lamps have been less accurately calibrated and are inherently more variable.

TABLE 10

Color Temperatures of Vacuum Tungsten Filaments at Various Efficiencies[a]

Lumens per Watt (Uncorrected)	Color Temperature (Uncorrected)	Lumens per Watt (Corrected)	Color Temperature (Corrected)
0.5	1644	0.58	1663
1.0	1777	1.14	1794
1.5	1866	1.70	1883
2.0	1939	2.26	1955
2.5	1998	2.82	2014
3.0	2050	3.37	2066
3.5	2096	3.93	2112
4.0	2138	4.48	2153
4.5	2175	5.02	2190
5.0	2208	5.57	2224
5.5	2241	6.12	2257
6.0	2269	6.66	2285
6.5	2299	7.21	2315
7.0	2327	7.76	2343
7.5	2354	8.30	2370
8.0	2380	8.85	2397
8.5	2406	9.39	2423
9.0	2431	9.94	2449

The first two columns show the lumens per watt and color temperatures as directly determined experimentally from a given lamp. The second two columns give these same quantities corrected for losses due to cooling effects of leading in and supporting wires and absorption of the lamp bulbs.

[a] Hyde, E. P., Cady, F. E. and Forsythe, W. E., Color Temperature Scales for Tungsten and Carbon, *Phys. Rev.*, (2), **10**, Table I, p. 401; 1917.

3. STANDARDS OF SPECTRAL TRANSMISSION

The characteristics of physical objects which determine their colors, when viewed by radiation from other sources, can be expressed almost completely by means of spectral reflection or transmission curves, representing as a function of wave-length or frequency, the fraction of the original radiation impinging upon the object, which finally leaves it as reflected or transmitted rays respectively. Such curves are most readily determined by means

of a spectrophotometer. For the intercomparison of the color values of objects, without reference to the radiation by which they are viewed or the observer's visual system, reflection and transmission curves are of great utility in colorimetrics, although such curves represent properties of objects rather than of immediate stimuli to color. However, in view of Hering's principle of "the color-constancy of visual objects" (**24**), representing the tendency of the visual processes to compensate for variations in spectral constitution and intensity of the illuminating source, these curves attain some direct significance for consciousness. Although colors due to selective reflection are of more common occurrence than those due to selective transmission, the latter are of greater scientific importance because of the far higher degree of selectivity which is obtainable by transmission than by reflection.

The spectral transmission distributions for a number of technically important materials are considered below.

A. STANDARD THREE-COLOR ADDITIVE FILTERS.—There are several common applications of the principle of matching or of reproducing colors by the mixture of two or three stimuli, of constant relative spectral constitutions but varying proportions, which utilize selectively transmitting radiation "filters." Although their transmissions may be varied within certain limits without deleterious effects, it is desirable to specify the transmission curves of certain of these filters which have been found satisfactorily to fulfill their purposes.

(*a*) *Trichromatic Analyzer Filters.* Table 11 gives the transmissions of three filters employed in the Ives colorimeter (**29; 30**), which is employed for the designation of the colors of materials in terms of three mixed elementaries, determined by these filters.

(*b*) *Photographic Taking Filters.* Table 12 records the transmissions of four filters designed for making color separation photographic negatives on panchromatic emulsions. Rigid standardization of filters for use in this connection is not possible on account of the variations in sensitiveness to radiation of different wave-lengths exhibited by these emulsions, but the filters specified in the Table have been found to give fairly satis-

factory results with emulsions of average character. Nos. 1, 2, and 3 constitute the tri-color set, while Nos. 1 and 4 can be used successfully for two-color taking. The choice of three-color taking filter transmissions should in general be such as to duplicate

TABLE 11

Transmissions for Various Wave-Lengths of Ives Colorimeter Filters[1]

mμ	Percentage Transmissions		
	Red	Green	Blue
400	.....	.....	26.0
410	.....	.....	26.0
420	.....	.....	25.9
430	.....	.....	25.9
440	.....	.....	24.9
450	.....	.....	22.0
460	.....	.....	17.5
470	.....	.....	11.0
480	.....	.9	6.9
490	.....	3.7	2.5
500	.....	7.8	1.2
510	.....	11.2	.3
520	.....	13.0	.....
530	.....	12.9	.....
540	.....	11.1	.....
550	.....	8.2	.....
560	.....	4.8	.....
570	.....	2.2	.....
580	.....	.8	.....
590	.....	.....	.....
600	.8	.....	.....
610	4.5	.....	.....
620	18.0	.....	.....
630	45.0	.....	.....
640	64.7	.....	.....
650	72.4	.....	.....
660	76.9	.....	.....
670	79.4	.....	.....
680	81.3	.....	.....
690	82.3	.....	.....
700	82.9	.....	.....
710	83.0	.....	.....

[1] The values given in this table were supplied by Dr. H. E. Ives. Ives colorimeter filters measured at the Bureau of Standards depart appreciably from the above specifications, as do those examined by E. C. Bryant, *Astrophys. J.*, 55, p. 9, 1922.

TABLE 12

Transmissions of Approved Photographic Taking Filters

λ = Wave-length in $m\mu$.

T = Fraction of incident radiation transmitted.

No. 1		No. 2		No. 3		No. 4	
λ	T	λ	T	λ	T	λ	T
700	.82	620	.01	400	.07	460	.02
680	.82	600	.03	410	.09	470	.09
660	.82	580	.18	420	.11	480	.19
640	.82	560	.30	430	.15	490	.35
620	.80	540	.46	440	.21	500	.49
610	.75	530	.53	450	.28	510	.58
600	.60	520	.57	460	.26	520	.59
590	.25	510	.52	470	.19	530	.55
585	.10	500	.35	480	.10	540	.49
580	.02	490	.12	490	.04	550	.38
.....	.....	480	.02	580	.01	560	.25
.....	.....	.....	.....	.....	.....	570	.74
.....	.....	.....	.....	.....	.....	580	.07
.....	.....	.....	.....	.....	.....	590	.02

photographically, as closely as possible, the mixing proportions of the corresponding reproducing filters which are required to match the colors of the photographed objects. The production of such photographic records of the values in question will evidently depend not only upon the transmissions of the filters but also upon the spectral distribution of sensitiveness of the given photographic emulsion. Similar general principles apply to the choice of two-color taking filters if the blue-violet values of the photographed objects are left out of consideration, since two-color taking filters are ordinarily selected so as to neglect those values of the scene which are poorly represented in yellowish, artificial illumination.

(*c*) *Photographic Reproducing Filters.* Filters necessary for three-color and two-color additive projection of photographic positives printed from color separation negatives, or for use in such instruments as the Ives Chromscope (**54**, 218) depend in quality upon the character of the light source which is employed. Since the sources vary widely, it is difficult to specify the transmissions of the filters very exactly. Filters for three-color addi-

tive reproduction should be so selected that mixtures of the transmitted radiations are capable of matching a maximal number of colors. This means that the radiations in question should evoke hues approximating as nearly as possible to spectral saturation, and so distributed in the color triangle that the lines joining them lie maximally close to the spectral locus. Two-color reproducing filters in practice usually resemble closely the red and green members of a three-color set but may have a somewhat reduced saturation, and the dominant hue of the green is usually shifted somewhat more towards the blue than is that of the red number.

B. STANDARD SOLUTIONS.—The spectral transmissions of dyes and inorganic salt solutions of known purity and concentration are in course of determination at the Bureau of Standards, and some of the most important of these will be presented in later Reports by the present Committee.[17]

C. LOVIBOND AND OTHER COLORED GLASSES.—Similar statements apply to the Lovibond glasses, which are widely used as technical standards in the ranking of oils and other materials as regards color. There are a very large number of these glasses, and the accuracy with which they are reproduced in different sets is often relatively low (**77**). The spectral transmissions of a considerable number of precision-made glass plates of various colors have been determined at the Bureau of Standards, and these plates can be borrowed under proper restrictions by those desiring to check their spectrophotometric equipment.

4. STANDARDS OF SPECTRAL REFLECTION

A. SUBTRACTIVE PIGMENT ELEMENTARIES.—A committee of the American Institute of Graphic Arts is at the present time working on the colors of inks for three- and four-color printing processes. Pending their findings the present Committee will offer no recommendations on this matter. As previously noted, the pigments or dyes required for the satisfactory rendering, by the subtractive method, of photographic "color separations" are in general the *physical complementaries* of the corresponding additive

[17] Three notable atlases of absorption spectra are (**39**), (**96**), (**58**).

reproducing filters, i.e., they transmit or reflect those portions of the spectrum which the latter absorb, and *vice versa*.

B. SYSTEMS OF PIGMENT STANDARDS.—There are now available several notable systems of reflection color standards (**6**). Those which are best known to American workers are the Munsell system (**61**) designed for use by artists and the Ridgway system (**87**) intended for the ornithologists. More recently Ostwald (**71**; **72**) in Germany has published an elaborate scheme of this sort. Each of these systems comprises pigments of various selective and total reflection spaced fairly evenly over the total field of possible pigment colors.

(*a*) *The Munsell System* is based upon ten hues and nine degrees of "value," or of light reflecting power. Each of the hues is represented at each level of reflectivity by as many different saturation steps as is feasible. The system as published includes a text on "Color Notation,"(**61**) an atlas, (**62**) and a color sphere, as well as cards embodying the separate pigment values. Priest and his associates (**83**) have determined the spectral reflection curves of representative members of the pigment system, and have made recommendations for its improvement, although in general commending it as it stands. Evaluations of certain Munsell colors in terms of elementary sensations are given in the next Part of the present Report. For the spectral reflections reference should be had to Priest's original paper.

(*b*) *The Ridgway System* utilizes thirty-six hues, having approximately equal spacing on the spectral chroma scale, each of these being diluted with white in three degrees and with black in three degrees, making 1115 colors in all. Jones (**42**) has made careful monochromatic analyses of the undiluted colors, and has determined the separations of their dominant hues on the hue scale, the average for the spectral hues being 3.5 and for the purples 4.4 just noticeable steps. Considering the practical difficulties encountered in the preparation and reproduction of pigment samples the gradation of values exhibits excellent uniformity. The results of Jones' analyses are given in Table 13. Spectrophotometric measurements on the Ridgway pigments

TABLE 13

Monochromatic Analyses of Certain Ridgway Colors[a]

1	2	3	4	5	6	7
No.	Name	λ(mμ)	Per cent. Hue	dλ	d_s	d_s
1	Spectrum Red....	633	55	.....	.....	.....
2		616	31	17	5.0	5.0
3		610	34	6	2.0	3.0
4		605	34	5	1.5	2.0
5		.....	.....	.....	2.0	2.5
6	Orange Chrome...	597	40	8	2.5	2.5
7		595	34	2	1.5	1.5
8		593	26	2	1.0	1.5
9		589	23	4	2.0	3.0
10		586	31	3	2.0	3.0
11		582	34	4	2.5	3.0
12	Lemon Yellow	579	29	3	3.0	3.0
13		577	34	2	2.0	2.0
14		574	39	3	2.0	2.0
15		569	42	5	3.0	3.0
16		566	42	3	2.0	2.0
17		548	47	18	6.0	8.0
18	Emerald Green....	521	63	17	9.0	9.0
19		518	66	3	2.0	2.0
20		510	63	8	3.0	3.5
21		495	55	15	10.0	10.0
22		490	54	5	4.0	5.5
23		486	51	4	3.0	3.0
24		484	73	2	1.0	1.5
25	Spectrum Blue....	479	70	5	2.0	2.5
26		476	57	3	2.0	2.0
27		470	62	6	3.0	3.0
28		460	62	10	3.0	4.0
29		445	58	15	5.0	8.0
30	Spectrum Violet...	425	60	20	7.0	10.0
Mean....	.....	.....	.....	.....	3.2	3.8
31		570	22	.....	.....	.....
32		559	15	11	5	6
33	Purples.....	555	13	4	2	2
34		538	16	17	4	5
35		534	11	6	3	2
36		512	9.4	22	7	8
Mean....	.....	.....	.....	.....	4.2	4.6

[a] The numbers in column 1 show the spectral order of the thirty-six undiluted Ridgway chromas. Column 2 gives the names assigned to certain of the colors by Ridgway. Column 3 shows the wave-length of the dominant hue and column 4 the per cent. hue. Columns 5, 6, and 7 list the differences between succeeding members in wave-length, Steindler "hue scale," and Jones "hue scale" units respectively (See Jones, Ref. 42, pp. 73–77).

are being made at the Bureau of Standards, and will be considered in later reports of the present Committee.

(*c*) *The Ostwald System*, besides having been elaborately described by its author in a number of publications, has been given two concrete exemplifications. The most elaborate one, "Der Farbenatlas," presents approximately twenty-five hundred colors which are systematically indexed in order of their color tone and content of black and white respectively. An abridged system, "Der Farbkörper," comprises seven hundred and sixty-eight of the principal colors used in the more comprehensive system. Ostwald's arrangement, like that of Munsell and Ridgway, is based upon psychological rather than physical criteria. Kohlrausch (**44**) has made a spectrophotometric analysis of sixty of Ostwald's colors taken from three of his color circles having different saturations or reflectivities. Kohlrausch has furthermore computed the excitation values of these colors on the basis of König and Dieterici's excitation curves. He has also made direct monochromatic analyses of the colors in question.

V. METHODS OF COLORIMETRY AND THEIR INTERRELATIONS

It is not our purpose in the present Report to consider in detail the various methods which are in use, or which have been proposed, for the practical measurement of color. This important task, which involves the description of instruments and the establishment of a terminology for each of the methods, is reserved for a later report. However, it is desirable here at least to catalogue the available systems of colorimetry, and to consider in a preliminary way some of the problems which arise in connection with them, especially that of the reduction of data obtained by the various methods to a common comparison basis.

1. RÉSUMÉ OF AVAILABLE METHODS

The color of an object, considered as an impression which the object produces on the observer, is determined by at least three general sets of factors: (1) the physical characteristics of the object, (2) the physical characteristics of the radiant energy falling upon or emitted by it, and (3) the nature and condition of the observer's visual apparatus. Our control over color in its

technical applications is confined almost exclusively to the first two sets of factors, although the ultimate goal is always to be found in the consciousness of some observer. Practical colorimetry is therefore concerned with means for the unambiguous designation of those properties of objects and radiation which determine color perception. Most of the means actually employed, however, utilize the visual apparatus of an observer as an essential element—in determining an equation of colors—and hence the results are frequently not independent of the nature and special condition of this apparatus. For this reason it is necessary as in photometry, that the observers should be tested as average and normal.

A. SPECTROPHOTOMETRY.—A method of specifying the physical characteristics of objects and samples of radiation, for the purposes of colorimetrics, which is actually independent of the observer is found in various applications of spectrophotometry or spectroradiometry. These devices enable us to establish the spectral distributions of reflection or transmission for objects, or of energy for radiation, and thus to specify perfectly the essential factors in their values as color stimuli. On the side of the stimulus, pure and simple, spectrophotometry is the fundamental method of colorimetric specification. All other methods (except spectroradiometry) fail to give an equally complete account of the stimulus characteristics. The excuse for their use, however, lies in the fact that the detail of spectral distributions is not actually required if our concern is solely with final color result produced in the observer's consciousness, and methods which dispense with this detail have the advantage of increased simplicity, both in practical application and in expression of results. (Since there is a special Committee of the Optical Society on the subject of Spectrophotometry, it can not be a function of the present Committee to consider this topic in detail.)[18]

Complete data based upon spectrophotometry specify the colorimetric value of a stimulus in terms of the identical radiant power actually evoking the color, both in regard to total amount and

[18] On Spectrophotometry in general see (45).

spectral distribution. Other methods accomplish this result in terms of a complex stimulus, having general characteristics varying with the method, but seldom spectrally identical with the stimulus to be measured, which nevertheless is found empirically

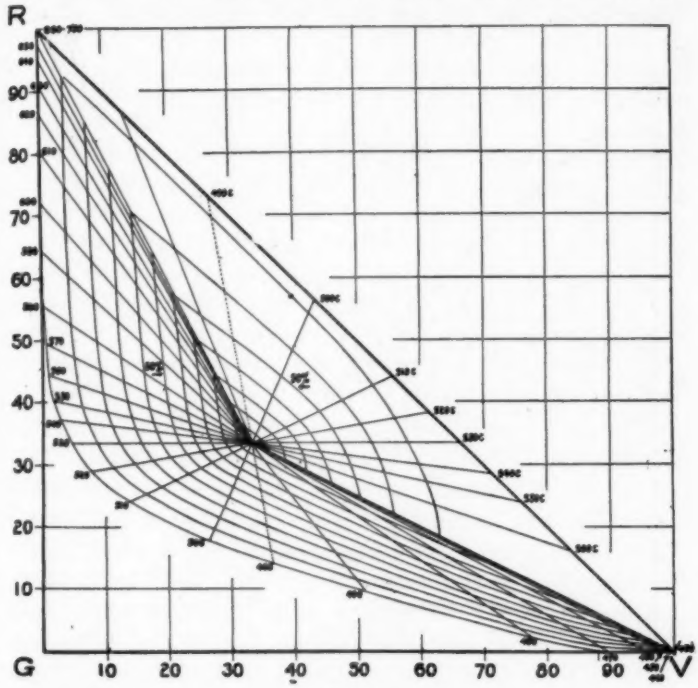

FIG. 9

Triangle, Showing Loci for Various Monochromatic Analyses
(The ordinates give the percentage excitation values for Red and the abscissae those for Violet.)

to evoke the same color as the latter. The most important of these methods which depend upon simple color matching are as follows.

B. MONOCHROMATIC ANALYSIS.—In this method the variable stimulus is composed of heterogeneous radiation, which by itself evokes white or gray, combined with homogeneous radiation of variable wave-length. The total intensity, ratio of homogeneous

to heterogeneous, and wave-length of the former, are adjusted until a match is obtained (66). The measured color is then designated in terms of (1) luminosity, (2) dominant wave-length, and (3) "per cent. hue." It is apparently not essential, although advisable, in this method that the heterogeneous radiation

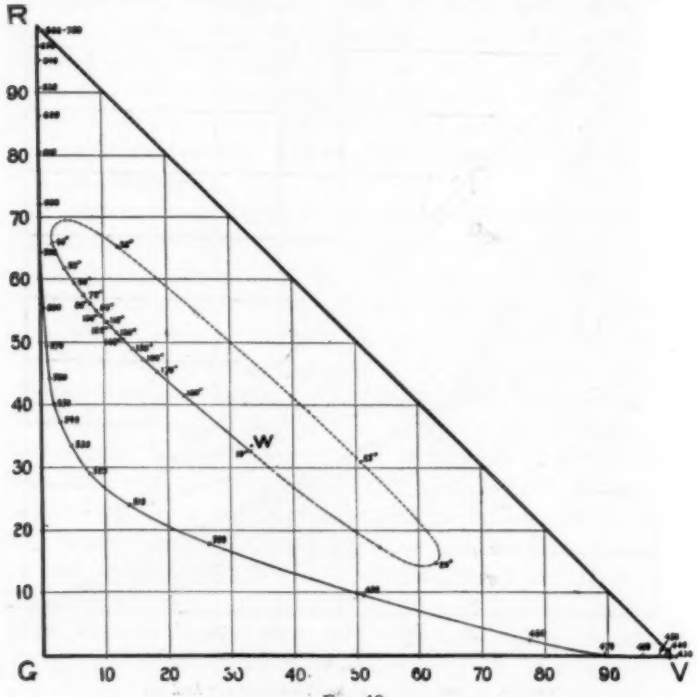

FIG. 10

Color Triangle, Showing Loci of Spectral and Certain Rotatory Disperson Excitations
(The ordinates give the percentage excitation values for Red and the abscissae those for Violet.)

should have an invariable constitution, e.g., that of "normal gray light," so long as it always produces an achromatic color with the given observer. Results obtained by this method are subject to serious errors with deviations, of common occurrence, of the observers' visual systems from the average normal. It is

important, therefore, that the observers should be carefully selected so as to have standard visibility and color excitation curves. The direct application of this method is limited to colors possessing spectral hues. In order to specify purples, it is necessary to determine the wave-length of homogeneous radiation which when mixed with them matches the heterogeneous standard, i.e., which yields a gray of the given brilliance. It is clear, however, that the method is capable of dealing with colors of all degrees of saturation.

C. TRICHROMATIC ANALYSIS. (**29; 30.**) In this method the variable stimulus is composed of three constituents having appropriate spectral constitutions, which yield, respectively, colors corresponding in hue to the two end, and the middle regions of the spectrum. The colors ordinarily chosen are red, green and blue, and to render the field of applicability of the system as wide as possible the three stimuli should each be maximally homogeneous. Relative spectral distributions within the components remaining constant, their proportions and total luminosity are adjusted until a match is obtained. The measured color can then be designated by three intensity or luminosity values, one for each of the three components. If the spectral distributions of the components are determined by fairly narrow-banded filters, this method is capable of dealing with practically all reflection colors, which are relatively unsaturated, but is not satisfactorily applicable in its simplest form to many saturated filter or spectral colors. In order to extend the method to deal with the latter it is necessary to add a variable quantity of "white" to the sample which is being measured. If this is done the colorimetric significance of the added white can be recorded in the resultant measurements by subtracting its amount from each of the three-color readings, yielding a specification in terms of red, green and blue, with one minus coefficient, a mode of expression which is in no way incompatible with the trichromatic principle. The trichromatic analyses are of fundamental interest on account of their maximally direct relation to the triadic response mechanism which apparently underlies all color vision,

but the results which are obtained vary with the color excitation curves of the given observer.

A new and promising application of the trichromatic method of color analysis is to be found in Jones' subtractive colorimeter (**43**). Although the psychophysical principles involved in this instrument are substantially the same as in three-color analyzers based upon additive mixture, the physical operation of the instrument is quite different from that of a three-color additive color-matching device. White or tungsten light is passed in succession through three wedge filters which absorb respectively red, green and blue, and an adjustment of the three wedges can be found which yields a color-match with the sample which is under examination. A neutral wedge is also provided in the path of the beam, permitting a match to be obtained by the use of only two of the chromatic wedges in combination with the neutral one.

D. ROTATORY DISPERSION SYSTEMS.—Several partial systems of color specification have been based upon the rotatory dispersion of quartz, the degree of this dispersion being a function of the thickness of the quartz plate which is employed (**76; 81**). By placing such a plate properly between nicol prisms or other polarizing devices, variations in the relative angular positions of the latter with respect to their extinction positions may be employed to determine a wide variety of spectral transmissions, all of which, however, follow a definite law. Still greater flexibility is obtained by the use of two quartz plates and three nicol prisms. A radiation source of known energy distribution is ordinarily utilized. Instruments based upon this principle are the Arons chromoscope and the leucoscope, recently studied very thoroughly by Priest. The principle appears to have promise of very wide applicability, especially as a means of producing and of specifying a large variety of spectral distributions which it is difficult or impossible to obtain with filters or original sources of radiation. In general, however, the method depends upon the matching of apparent colors, rather than of identical energy distributions.

E. PLANCKIAN DISTRIBUTION ANALYSIS.—The series of spectral distributions determined by successive values of T in Planck's

equation (see page 556) for the radiation emitted by a "black body" provide a system of stimuli, easily specifiable, and evoking a characteristic series of colors (including gray).[19] However, since the system is virtually unidimensional it can only be applied to a very limited range of colors, practically only to those due to stimuli whose relative spectral distributions fit the Planckian equation. The value of T required for the black body to produce the color match is used as an index of the color, being called the "color temperature."

F. COMPARATOR METHODS.—Methods of color specification based on color matching with arbitrary standards are at present of great technical importance. Such standards include selectively transmissive solutions of definite composition, as well as colored glasses—as in the Lovibond Tintometer—and variegated pigments—as in the Munsell, Ridgway, and Ostwald systems. The final results of measurement by means of one of these methods are expressed in terms of a number or numerical symbol, standing for the particular standard which most nearly approximates the sample in color. These devices are simple in their practical applications, but tend to be unreliable and inaccurate, while the results obtained by different systems are difficult of intercomparison.

2. THE INTERCONVERSION OF DIVERSE COLOR SPECIFICATIONS

One of the main interests of the present Committee is to provide means by which color specifications in terms of different systems can be reduced to a common denominator and, so far as possible, be interconverted (**36**). Spectrophotometric data are potentially convertible into the data of any other system whatsoever, but no specifications which are based upon simple color-matching can be reduced to spectrophotometric terms, without additional information. However, a satisfactory common denominator for all systems is apparently provided by the *elementary color excitations*. Values of these excitations can be found which will specify completely the color characteristics of any stimulus, and each

[19] Use of color temperature as a means of color specification has been developed extensively by E. P. Hyde and his collaborators (**28**).

member or possible specification in every color system can be reduced to such excitation values, and hence can be assigned a certain position on the color-mixture triangle. In this way the data of separate systems can be definitely intercompared, and can be interconverted in so far as the representations of the several systems overlap; with the obvious restriction that peculiarities of the stimulus—such as spectrophotometric details—which determine no characteristic excitation values, are necessarily lost.

It would therefore appear that the first step in our task is to provide means for transforming the data of each colorimetric system into elementary excitation values, and where possible, means for the reverse transformation. When such transformations have been made, it will be easy to determine the equivalents of one system in terms of any other system. The general principles underlying these computations have already been outlined briefly in our presentation of the excitation curves (*vide supra*). The spectral energy distribution of a given standard stimulus is required if the latter is to be dealt with directly, but can be dispensed with as soon as its combination with the elementary excitation curves has provided a specification of the stimulus in terms of the elementaries.

A. SPECTROPHOTOMETRIC DATA TO EXCITATIONS.—Spectrophotometric data are usually given in the form of spectral transmission or reflection curves. Such curves require combination with a certain energy distribution—representative of the particular source by which the object is viewed—in order that they should become determinative of a definite color. The process of reducing any given spectrophotometric specification to excitation values is therefore as follows. (a) Multiply each of the ordinates of the transmission or reflection curve by the corresponding ordinates of the energy distribution curve of the source. (b) Multiply each of the ordinates of the resulting curve by the corresponding ordinates of each of the color excitation functions as given in Table 6 (under "Excitations"), this being a separate operation for each of the three excitations, yielding three separate curves which represent the respective excitation values for each wave-

length of the given stimulus. (c) Determine separately the areas of the three curves thus found. This latter operation can be performed by applying a planimeter to a graph of the resultant curves or—with sufficient accuracy—by finding the sum of representative ordinates of each curve, taken separately at uniform, small, intervals—such as 10 $m\mu$—throughout the range of the curve in question. (4) Reduce the three areal values thus obtained to percentage form, so that their determined ratio remains unchanged but their sum becomes equal to 100. The color excitation values can now be expressed by means of two numbers, representing the red and violet excitation percentages, that for the green being obtainable by subtracting the sum of these two values from 100.

As already pointed out, it is in general impossible to reverse the above process, and to convert color excitation specifications into definite spectrophotometric form, because there are an infinite number of spectrophotometric conditions for the majority of color excitation ratios. However, it is possible by means of the color triangle to determine stimuli for given sets of excitation values. The most feasible method of procedure is to plot the position of the given color in the triangle and to note its relation to the locus of the spectral colors. If it lies outside of the area bounded—on two sides—by this locus it possesses no realizable stimulus. If it lies exactly on the locus, in a region of the latter which exhibits curvature, it possesses a unique condition, viz., the homogeneous spectral stimulus having a wave-length indicated by its position with respect to the wave-length scale plotted on the spectral locus. If it falls on a straight portion of the locus in question, it can be evoked by the homogeneous wave-length which immediately corresponds with its position, or by mixtures of any stimuli having wave-lengths represented on either side of it in the given straight portion of the locus, the proportions of these mixed stimuli being determined by the "center of gravity" principle (*vide supra*). If the point representing the given color lies *within* the area bounded by the spectral locus, the color can be produced by mixtures of spectral stimuli lying at the intersections, with the locus, of any straight line passing through the

point in question, the proportions being determined again by the "center of gravity" principle, applied to the segments of the line thus established. There are obviously an infinite number of such mixtures, not only of two components but of any number of components.

It is one of the functions of the present Report to provide the excitation values of characteristic stimuli, computed by the method outlined above. Tables of such values, for black body colors, colors obtained by rotatory dispersion, Munsell colors, etc., will be found below in conjunction with the discussion of the excitation equivalents of these various standards.

B. MONOCHROMATIC ANALYSIS DATA TO EXCITATIONS.—The general principles underlying the reduction of monochromatic analysis specifications to color excitation values are similar to those outlined above, but with certain complexities which are introduced by the use of a photometric method for establishing and expressing the ratio between the amounts of "white" and monochromatic stimulus in any given case. It is of course natural in practice to specify this ratio in luminosity terms, but these terms play no part in determining the excitation values given in Table 6. Consequently, in order to effect the requisite transformation, it is necessary to make use of the luminosity valencies of the several excitations, which were discussed on page 551. To simplify computation, these valences have been expressed so as to represent the fractional contributions of the three excitations to the luminosity of a white, taken as unity, the values being: for the red 0.370, for the green 0.617, and for the blue 0.012.

The actual steps which are involved may be outlined as follows. (*a*) It is first necessary to know the spectral distribution of radiant intensities for the stimulus which is employed as a white in the given measurements. In case the distribution in question is that of average noon sunlight—or a distribution which color-matches this—it is only necessary to multiply each of the luminosity valences above considered, by the "per cent. white" of the specification. If, on the other hand, the "white" departs in effective character from average noon sunlight, excitation values must be computed for it by the method described under

"A" above. Each of these values is then multiplied by the corresponding luminosity valence, and the products thus obtained are reduced, with ratios unchanged, so that their sum is equal to unity. These figures are now multiplied separately by the percentage measure specified for the "white." Either one of these operations—for average noon sunlight or the arbitrary "white"—yields a set of three figures, one for each of the excitations. (*b*) The next step is to treat the "per cent. hue" measure in a similar manner. The excitation values for the wave-length employed in the given match must first be looked up in Table 6. Each of the values thus found is next multiplied by the corresponding luminosity valence and the products are reduced so that their sum is equal to unity. Each of the resulting values must now be multiplied by the percentage measure of the monochromatic component in the original specification. (*c*) The corresponding members of the two sets of values, thus secured,—for the "white" and "hue" respectively—are now added. (*d*) The three resulting sums express the excitation values of the monochromatic specification in luminosity terms. In order to reconvert them into the color valence terms of Table 6, each of these sums must be divided by the corresponding luminosity valence, and the values thus obtained reduced to the usual percentage form.

In the case of specifications by monochromatic analysis, of colors possessing a purple hue, in terms of per cent. of the given color and the per cent. of its spectral complementary required to be mixed with it to match the standard white, the procedure for reduction of the data to excitation values differs from the above in the following way. (*a*) The "white" is treated exactly as described under "(*a*)" of the preceding paragraph, except that the final percentage employed as a multiplier is 100. (*b*) The complementary monochromatic stimulus is treated exactly as under "(*b*)" in the same paragraph, the final percentage multiplier representing the per cent. which this stimulus is of the mixture comprising it and the measured color. (*c*) The individual members of the set of values thus obtained for the complementary stimulus are then *subtracted* from the corresponding members of the set obtained for the white. (*d*) The three resulting differ-

ences may now be reconverted into color valence terms, as directed under "(*d*)" of the preceding paragraph.

At the present time, owing to the unreliability of the magnitudes assigned to the luminosity valences for the three excitations, the conversion of data obtained by monochromatic analysis into excitation values, and thence into terms of other methods of color specification, cannot be accomplished with as great an accuracy as could be wished for. Present indications, moreover, are that these luminosity valences vary considerably among individual observers, without parallel variations in the color valences. Such variations evidently accompany deviations in the form of the observer's visibility curve from normal, and demand that special care be taken in the selection of observers for use of the monochromatic method. In general, this method would appear to be more sensitive to the personal equation than the trichromatic and certain other methods.

The reverse conversion, of color excitation values into monochromatic specifications, is theoretically possible without ambiguity for all sets of values represented in the color triangle by points lying within the area determined by the spectral locus (*vide supra*). The easiest means for accomplishing this conversion consists in the use of a color triangle having represented upon it not only the spectral locus, but also the loci of the spectral colors, and purples, mixed with various proportions of white. In the absence of such a diagram, which is provided by Fig. 9, a cumbersome "trial and error" method is necessitated. To determine the monochromatic equivalents of any color excitation specification, its position should be plotted on the color triangle of Fig. 9, and a straight line drawn through this point and the point representing the white (the white of the given monochromatic system). The intersection of this line with the spectral locus will indicate the dominant hue or wave-length,—direct or complementary, as the case may be—, and the relation of the color point on this line to its intersections with the loci for various percentages of admixed white will serve to determine the "per cent. white."

Table 14 (A and B) gives the excitation values computed for a large number of representative monochromatic specifications, which were read off from Fig. 9. It should be borne in mind that all of these values are subject to correction with improved determinations of the luminosity valences.

C. TRICHROMATIC DATA TO EXCITATIONS.—Data obtained by the trichromatic method of analysis bear the most direct possible relation to color specification in terms of elementary excitations. However, no actual colorimeter based on this principle can duplicate in saturation the elementaries which were employed in computing the values of Table 6, at least in the case of the green excitation, and probably also in the cases of red and violet excitations. Extant three-color measuring systems naturally

TABLE 14B

Color Excitation Values for Representative Monochromatic Analyses; Purple Hues

Complementary Wave-Length		Per cent. Hue						
		10	20	30	40	50	60	70
495C	R	35.5	38.0	41.2	45.3	50.2	57.8	69.6
	V	32.9	32.5	32.0	31.3	30.5	29.2	27.3
500C	R	34.7	36.4	38.4	40.8	44.0	48.2	54.2
	V	33.8	34.6	35.5	36.6	38.0	39.8	42.5
510C	R	34.0	34.9	35.8	37.0	38.4	40.2	42.5
	V	34.7	36.6	38.6	41.0	44.0	47.8	52.6
520C	R	33.7	34.0	34.5	35.0	35.6	36.3	37.3
	V	35.0	37.2	39.8	42.6	46.2	50.5	56.0
530C	R	33.3	33.3	33.3	33.3	33.3	33.3	33.3
	V	35.3	37.7	40.5	43.6	47.4	52.3	58.1
540C	R	33.0	32.7	32.3	31.9	31.4	30.8	30.0
	V	35.5	38.0	41.0	44.2	48.3	53.5	59.9
550C	R	32.7	32.2	31.6	30.8	30.0	28.8	27.3
	V	35.6	38.2	41.3	44.7	48.8	54.2	61.0
560C	R	32.4	31.6	30.5	29.3	27.8	25.9	23.4
	V	35.8	38.4	41.7	45.3	49.6	54.9	62.2

TABLE 14A

Percentage Color Excitation Values for Representative Monochromatic Analyses: Spectral Hues.

Wave Length of Dominant Hue	Per Cent. White																			
	0		10		20		30		40		50		60		70		80		90	
	R	V	R	V	R	V	R	V	R	R	R	V	R	V	R	V	R	V	R	V
400	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
410	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
420	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
430	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
440	0	99.3	.3	98.7	.6	97.9	1.0	97.2	1.5	96.2	2.5	94.5	3.4	92.6	4.7	90.0	7.4	84.5	13.0	74.0
450	0	98.3	.3	97.8	.7	97.0	1.2	96.0	1.8	94.7	2.8	92.8	4.0	90.5	5.5	87.5	8.5	81.5	14.0	71.0
460	0	95.7	.5	94.9	1.0	94.0	1.7	92.5	2.6	90.9	3.9	88.4	5.5	85.5	7.3	82.0	10.8	75.5	17.6	63.0
470	0	89.6	.9	88.2	2.0	86.4	3.1	84.5	4.6	81.9	6.3	78.8	8.7	74.7	11.8	69.4	16.3	62.2	22.7	51.0
480	2.3	77.6	3.8	75.5	5.3	73.3	7.2	70.5	9.2	67.5	11.6	64.2	14.6	60.0	18.0	55.0	21.9	49.4	27.0	42.5
490	9.5	51.1	11.5	49.6	13.6	48.0	15.8	46.4	18.0	44.7	20.3	43.0	22.6	41.2	25.2	39.3	27.9	37.4	30.5	35.4
500	17.8	26.4	19.6	27.2	21.3	28.0	23.0	28.8	24.8	29.5	26.5	30.2	27.8	30.8	29.3	31.4	30.6	32.0	32.0	32.7
510	24.0	13.8	25.3	16.4	26.5	18.9	27.6	21.1	28.6	23.2	29.5	25.2	30.4	26.9	31.2	28.7	31.8	30.3	32.5	31.8
520	29.0	7.6	29.6	11.2	30.2	14.5	30.7	17.5	31.2	20.3	31.6	23.0	32.0	25.3	32.4	27.4	32.7	29.5	33.0	31.4
530	33.3	4.7	33.3	8.6	33.3	12.4	33.3	15.9	33.3	19.0	33.3	21.7	33.3	24.4	33.3	26.9	33.3	29.2	33.3	31.3
540	37.1	2.8	36.6	7.2	36.1	11.2	35.7	14.9	35.3	18.2	34.9	21.2	34.5	23.9	34.1	26.5	33.8	28.9	33.5	31.2

TABLE 14A—*Continued*

Wave Length of Dominant Hue	*Per Cent. White*																			
	0		10		20		30		40		50		60		70		80		90	
	R	V	R	V	R	V	R	V	R	V	R	V	R	V	R	V	R	V	R	V
550	40.2	1.7	39.3	6.2	38.4	10.3	37.5	14.1	36.7	17.6	36.0	20.8	35.5	23.6	34.9	26.3	34.3	28.8	33.8	31.1
560	44.2	1.0	42.6	5.5	41.2	9.6	40.0	13.5	38.8	17.2	37.7	20.4	36.7	23.3	35.7	26.0	34.9	28.6	34.1	31.0
570	49.5	0.7	47.2	5.2	45.0	9.3	43.2	13.0	41.4	16.6	39.8	20.0	38.3	23.0	36.9	25.8	35.6	28.5	34.4	30.9
580	55.4	0.4	52.5	4.7	49.7	8.8	47.2	12.6	44.7	16.2	42.3	19.6	40.3	22.7	38.4	25.5	36.6	28.2	34.9	30.8
590	64.3	0.0	60.4	4.3	56.6	8.4	53.0	12.3	49.7	15.9	46.3	19.3	43.4	22.4	40.6	25.3	38.2	28.0	35.7	30.7
600	72.2	0.0	67.2	4.1	62.6	8.1	58.1	12.0	53.9	15.6	50.1	19.0	46.2	22.1	42.7	25.1	39.5	27.9	36.3	30.6
610	80.4	0.0	74.7	4.0	69.1	7.9	63.8	11.7	58.9	15.3	54.0	18.7	49.6	21.8	45.2	24.8	41.1	27.7	37.2	30.5
620	86.3	0.0	80.0	3.9	74.0	7.7	67.8	11.5	62.3	15.0	57.1	18.4	52.0	21.5	47.1	24.6	42.4	27.5	37.8	30.4
630	90.8	0.0	84.1	3.9	77.6	7.6	71.4	11.4	65.3	14.8	59.6	18.2	53.9	21.3	48.5	24.4	43.3	27.4	38.2	30.3
640	95.1	0.0	88.0	3.9	81.2	7.5	74.6	11.3	68.2	14.6	61.9	18.0	55.7	21.2	50.1	24.2	44.3	27.3	38.7	30.3
650	97.5	0.0	90.2	3.8	83.0	7.4	76.1	11.2	69.4	14.5	62.8	17.8	56.5	21.1	50.4	24.1	44.5	27.2	38.8	30.2
660-700	100.0	0.0	92.4	3.8	85.0	7.4	77.9	11.2	70.9	14.4	64.1	17.7	57.6	21.0	51.2	24.0	45.0	27.1	39.1	30.2

differ also in the dominant hues of the three components which they employ. In order to convert the data obtained by the application of any such system to terms of our three elementaries, it is necessary to employ nine coefficients which represent the degree of participation of each of our elementaries in each of the components of the given system or vice versa. The reverse conversion involves nine reciprocal coefficients based upon the same relationship. The operations involved may be represented by the following equations:—

$$(1)\quad R = ar + bg + cv$$
$$(2)\quad G = dr + eg + fv$$
$$(3)\quad V = hr + ig + jv$$

where a, b, c, etc., are the coefficients in question, r, g, and v are the values of the given trichromatic measuring system, and R, G, and V are the desired excitation values.

TABLE 15

Coefficients for Interconverting Ives Colorimeter Data and Excitation Values

If r, g, and b are the components of a color according to the Ives colorimeter, and R, G, and V are its values in terms of the elementary excitations used in this Report;

$$R = 1870\,r + 2080\,g + 14\,b$$
$$G = 134\,r + 3710\,g + 124\,b$$
$$V = 506\,g + 3460\,b$$

Conversely:

$$r = 1275\,R - 719\,G + 21\,V$$
$$g = -46\,R + 646\,G - 23\,V$$
$$b = 7\,R - 95\,G + 665\,V$$

There is an arbitrary factor in all of the above coefficients, so that the results are significant only as proportionalities.

The only trichromatic additive system which is in any way well known at the present time is that employed in the Ives colorimeter. Table 15 provides values for the coefficients which must be employed to convert color specifications in terms of the Ives system into excitation terms by use of the formulae given above. These coefficients were obtained by applying the excitation data of Table 6 to the spectrophotometric curves for the filters and light source employed in the Ives colorimeter in accordance with the principles outlined under Section A of the present part of this report. The light source assumed was average noon sunlight.

Table 15 also gives coefficients for the converse operations. Similar methods of calculation may be applied to any three-color system, such as, for example, that involved in the new subtractive colorimeter designed by Jones (43). In the special case of the subtractive colorimeter, however, it is not possible to assign fixed excitation values to the three primaries of the instrument, since the spectral distributions vary qualitatively for each individual setting of the instrument. The resulting necessity of applying the principles for the interconversion of colorimetric data to the spectrophotometric analyses of individual settings is a theoretical demerit of the subtractive as opposed to the additive trichromatic colorimeter.

It is clear that in order to compare the data obtained by trichromatic analysis with those due in any other system, such as the monochromatic, it is only necessary to convert both sets of data into color excitation values, in which condition they may be translated by a further operation into terms of any desired system.

D. ROTATORY DISPERSION DATA TO EXCITATIONS.—Colorimetric data based upon any rotatory dispersion system may be converted into color excitation terms by determining the spectrophotometric curves for the given dispersion stimuli and applying the methods outlined under A above. The reverse conversion is best accomplished by means of a plot in the color triangle of the loci for the various series of dispersion colors which are involved. Such reverse conversion is of course possible only when the given excitation values determine a point in the triangle which falls upon one of these loci, although the great flexibility of the rotatory dispersion method will permit the duplication of a wide variety of conditions of color excitation.

Table 16 gives the excitation values for the rotatory dispersion colors produced at various angles of the Nicol prisms for a quartz plate one mm in thickness. These values are plotted in the color triangle in Fig. 10. It will be seen that their locus is approximately elliptical in form and corresponds very closely with that of a certain range of black body colors (Cf. Fig. 3).

TABLE 16

Excitation Values of Certain Rotatory Dispersion Colors

(Comparison source = acetylene color)
(Quartz thickness = 1 mm.)
(Φ = the angle between the Nicol prisms)

Φ	Per cent. *R*	Per cent. *V*	Nearest Black Body Color Temperature
0°	41.3	22.7	3494°
10°	32.6	33.1	5050°
20°	14.8	62.6	.....
30°	65.1	12.3	.....
40°	65.8	1.8	.....
50°	61.7	3.7	1690°
60°	59.2	5.3	1840°
70°	57.5	6.8	1970°
80°	56.1	7.9	2070°
90°	55.0	8.9	2160°
100°	54.1	9.7	2227°
110°	53.2	10.5	2302°
120°	52.4	11.2	2370°
130°	51.5	12.1	2448°
140°	50.4	13.1	2543°
150°	49.0	14.4	2673°
160°	47.2	16.2	2850°
170°	45.1	18.4	3050°

TABLE 17

Percentage Excitation Values for Black Body Colors Computed by Means of the Planckian Formula ($C_2 = 14{,}350$)

Degrees Kelvin		0	100	200	300	400	500	600	700	800	900
1000	R	78.1	75.4	73.0	70.7	68.4	66.3	64.3	62.4	60.6	59.0
	V	0.6	0.8	1.2	1.7	2.3	3.0	3.8	4.7	5.6	6.5
2000	R	57.4	56.0	54.6	53.4	52.2	51.0	49.9	48.8	47.8	46.8
	V	7.5	8.5	9.6	10.6	11.7	12.8	13.8	14.9	16.0	17.0
3000	R	45.8	44.9	44.0	43.2	42.4	41.6	40.8	40.2	39.7	39.0
	V	18.1	19.2	20.2	21.2	22.2	23.1	24.0	24.8	25.7	26.5
4000	R	38.4	37.8	37.4	36.8	36.3	35.8	35.3	34.9	34.5	34.1
	V	27.2	27.9	28.6	29.3	30.0	30.7	31.3	31.9	32.4	33.1
5000	R	33.7	33.3	32.9	32.6	32.3	32.0	31.7	31.4	31.1	30.8
	V	33.8	34.3	34.8	35.3	35.8	36.2	36.7	37.2	37.6	38.1
6000	R	30.6	.....	.....	.....	7000	28.4	.....	.....	.....	.....
	V	38.5	.....	.....	.....	.....	41.9	.....	.....	.....	.....

E. PLANCKIAN DISTRIBUTION DATA TO EXCITATIONS.—The various black body colors can evidently be reduced to color excitation values by means of their spectrophotometric representations, utilizing the methods already outlined. The reverse conversion is best accomplished by means of a plot of the positions for these colors in the color triangle.[20]

Table 17 gives the color excitation values for a wide range of black body temperatures and these values are represented graphically in Fig. 3.

F. COMPARATOR DATA TO EXCITATIONS.—The conversion of color specification data, based upon color-matching with arbitrary standards, to color excitation terms will of course involve not only the spectral transmissions or reflections of the given standards but also the spectral distribution curve of the particular radiation source which is employed in making the given color-match. The reverse conversion is best accomplished by means of a representation in the color triangle of the values for the various arbitrary standards, and the possibility of such reverse conversion

TABLE 18

Percentage Excitation Values for Certain Munsell Colors[a]

D	*E*	*D*	*E*	*D*	*E*
R 7/5	39R 29V	R 5/5	44R 25V	R 3/2	40R 28V
Y 7/4	43R 19V	Y 5/5	47R 14V	Y 3/2	40R 23V
G 7/4	34R 29V	G 5/5	33R 26V		
B 7/4	30R 37V	B 5/5	26R 42V		
P 7/3	32R 36V	P 5/5	29R 43V		

[a] "D" is the designation of the given color in the Munsell System and "E" is the corresponding percentage excitation value.

[20] It is clear that since the black body colors form a single linear series the reverse conversion will seldom be possible.

will of course depend upon the scope of the given comparator system and in general will be only approximate.

In Table 18 will be found color excitation values for various Munsell pigments as seen under average noon sunlight. The equivalents of certain Ridgway colors in terms of monochromatic analysis have been given in Table 13. As already noted, the excitation equivalents of many colors in Ostwald's pigment system have been computed very carefully by Kohlrausch in a recent article. In subsequent reports the committee will endeavor to provide values for further standards, both in these two and other systems, such as that utilized in the Lovibond tintometer.

VI. SUMMARY AND CONCLUSION

The above report, being a more or less pioneer effort of its kind, must naturally be regarded as incomplete and tentative. However, the purpose of the report is an earnest one and is directed towards at least four ends; (1) the clarification and standardization of color terminology, (2) the compilation of data which are fundamental to color science, (3) the specification of standard stimuli and conditions, for use in practical color work, and (4) the encouragement of discussion and research along these lines. An outline of the contents of the present report is given at the beginning in the form of a Table of Contents. It is hoped in later reports by the present Committee to deal more specifically with details in the terminology and application of the various methods for colorimetry as well as with the design of instruments for the utilization of these methods. Finally, the Committee desires to express once more its wish that workers in the field of color science communicate freely their criticisms and specific needs.

NOTE: No attempt is made here to summarize the progress in color science during the years, 1920–21, since this topic has been treated by the Chairman of the Committee in general summaries published in the *American Journal of Physiological Optics*, October 1921, pp. 316–391, and forthcoming.

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HETEROMORPHIES DUE TO THE VARIATION OF EFFECTIVE APERTURE AND VISUAL ACUITY[1]

BY
K. HOROVITZ

1. By the use of optical apparatus the image of the surrounding space sometimes undergoes a complete change, as in some kinds of prisms and mirrors. But also in other cases, in which the instrument produces a perfect image, apparent alterations of space are perceptible. These apparent defects of the visual space are called heteromorphies.

Expert microscopists are always surprised that beginners are unable at first to find the image, or, when drawing it, always make it too small. And to each of us it is well known that the drawing or photograph of a microscopical object seems much bigger than really shown by the microscope. Impressions of this kind are also perceptible in other cases than when using an optical instrument. If, for instance, instead of through the microscope we look through an empty tube with one eye and fix it on a distant object[2] while the other eye is directed to the same object in the ordinary manner, the image seen through the tube appears much smaller than to the naked eye.[3] Exactly the same thing can be observed by looking at a landscape through the finder of a camera or by holding a narrow stop before the eye. In all these cases the aperture of the rays of the optical system, consisting of the eye together with the effective stop, is changed.

The phenomena described can be observed by persons with normal vision (emmetropes), under certain conditions to be described later, by ametropes and, as already mentioned, in using optical instruments. The phenomena are therefore, as it seems, independent of the dioptrical properties of the system, but only

[1] Communicated by Dr. Ludwik Silberstein.

[2] It must be a distant object in order that the rays appear to come from infinity in the same manner as when using the microscope.

[3] Cf. St. Meyer, Phys. Zs. 21, p. 124, 1920; and K. Horovitz, Phys. Zs. 21, p. 499, 1920.

so far as the dioptrical system remains unchanged and the image remains clear.

2. The effective aperture may influence the image in different ways. When the entrance-pupil is altered, the size of the image-forming diffraction-disks is also altered. The smaller the latter are, the sharper the images, but their brightness is reduced (case of a small diaphragm). A stop which reduces the aperture of the image-forming pencils without altering the entrance-pupil, changes the field of view. The image of the stop in the focal plane (in the case of the eye, on or before the retina) is the exit-window (case of the tube and the finder of a photographic camera).

An alteration of the entrance-pupil changes also the actual size of the image, for the circles of confusion become smaller. As the aberrations in the eye and the formation of the image by wide-angle pencils bring it about the size of the diffraction-disks depends not only on the size of the entrance pupil, this reduction of the image on the retina is difficult to perceive. Thus the diminution observed, when looking through a diaphragm, need not depend necessarily on the actual reduction of the image on the retina. When, the entrance-pupil remaining constant, the entrance-window or the stop of the field of view is changed, the structure of the image is only changed in those places where now, instead of the former images, the image of the exit-window is formed. If nevertheless the impressions brought about by the unchanged retinal images are changed, evidently this must be connected with the mutual influence of neighboring parts of the retina,[4] especially with the influence of the simultaneous contrast-sensibility which brings about a change of the adjustment of the eye. Experiments have shown, that the diminution is as much increased as the stop of the field of view decreases (that is to say, in the case of the tube it would be a longer one). A diminution of the field of view also occurs with the replacement of the binocular by monocular vision, which transition also is connected with an apparent diminution of size.

[4] This is Hering's induction.

But an alteration of the aperture is also connected with an alteration of the distribution of light in the image. This is caused by a change of the circle of confusion by variation of the entrance-pupil. We have, further, to consider a change of the simultaneous contrast, when, by an alteration of the stop of the field of view, parts of the image are covered either by light or darkness. The former case occurs when an object is observed through a glass-tube, which, by total reflection, forms on the retina a brightly illuminated background; the latter when looking through a tube blackened on the inside, in which case the part of the field of view around the retinal image is darkened. But this affects in a decisive way the acuteness of vision and this influence therefore must always be taken into account.

3. It is well known, that the acuteness of vision is diminished by a reduction of the intensity of illumination, by flooding the retina with useless light, by the darkening of simultaneous contrast and of course also by wrong adjustment.[5] All these produce a reduction of the relative differences in the sensations of the brightness of two points. An increase of the visual acuity is caused by: a diminution of the entrance-pupil, a moderate increase of the intensity of light (provided that the peripheral portions of the retina are not sensibly illuminated by stray light), and the illuminating simultaneous contrast. Each change of the incident light alters the pupillary diameter leading to disturbing secondary phenomena (the acuteness of vision may be changed thereby and also focusing movements be liberated).[6] Therefore, it was necessary to make experiments in which the influence of the size of the pupil was eliminated. For this purpose investigations were made on persons, whose pupils had lost the reaction to light. *The result was, that a reduction of the acuteness of vision is always followed by an apparent diminution in size and, at the same time, the removing and bringing nearer together of the objects observed.* On the other hand a sudden increase of the visual acuteness produces

[5] In the latter case also dioptrics may be of importance as mentioned before.

[6] Cf. E. Hummelsheim, Arch. f. Ophth. vol. 45, 1898, p. 357 and K. Horovitz, Ber. d. deutschen physik. Ges. 1921, 2, pp. 9–11 and Sitz. Ak. Wiss. Wien, vol. 1130, 1921, pp. 405–421.

an increase of size. With atropinisized ametropes it could be observed that a small (stenopäical) stop which in the case of normal sight would cause a diminution, improves the acuteness of vision so much that an increase may be observed. Beside this *new* effect of the alteration of size in the field of vision, the known influence of the visual acuity must also be considered and also the influence of the aperture on the depth of focus of the image-space. The importance of the acuteness of vision for the resolving power of the microscope was pointed out by F. E. Wright.[7] (Here the illuminating effect of the simultaneous contrast comes into play.) In the following I intend to deal with a series of contrivances, for which the above mentioned effects are of importance, and then set forth the theory of the phenomena.

4. For the series of readings a simple lens is used, before which a diaphragm is placed, to increase the distinctness. With a very small diaphragm (e.g., of 0.1 mm. diameter) the usual magnification cannot be observed. I made this observation myself with an Elster-Geitel-electroscope and found it confirmed by other observers: the scale and the leaves of the electroscope seem to be in one plane and far more distant than without a diaphragm. As these phenomena are not always perceptible with both eyes with the same intensity they are also of importance when first one eye is used and then the other in observing and above all in comparing the images with one eye aided and the other naked. This is the case with the well-known examination of the magnifying power of a telescope which consists in comparing the lines of a distantly suspended scale with one eye naked and the other aided by the telescope. If then the exit-pupil of the telescope is much smaller than the pupil of the eye, the phenomenon mentioned above is observed. Thus sometimes it is not sufficient to pull out the tube to diminish the parallax between the two images. A similar examination of the magnifying power of a microscope is open to the same difficulties. To avoid such faults, fix close to the instrument, whose magnifying power is to be examined, a diaphragm for the naked eye of the size of the exit-pupil of the

[7] F. E. Wright, *Jour. of the Opt. Soc.* 2-3, p. 101, 1919.

instrument. The right perception of the increase, when using night-glasses depends also on the correct proportion between the exit-pupil of the instrument and the eye-pupil. A. Gehlhoff[8] pointed out that the size of the field of view is also of psychological importance for the resolving powers of these instruments. By employing drawing apparatus, as for instance the well-known camera lucida, the influence of the opening of the stop is remarkable. On using this instrument the scenery appears smaller on the drawing paper than to the naked eye. *If the stop is made gradually smaller* (as far as it is possible without making the illumination of the image so faint, that it is no longer perceptible) *the image becomes smaller as well.* Qualitative tests, made with regard to this, have shown that the variation in size by stopping down the diaphragm from 3 mm to 0.1 mm is about 15%.

It is natural to take into account the variation of visual acuity by dazzling. Therefore every physicist or astronomer uses in exact measurements a dark eye-shield to cover the non-observing eye. Also blinding of this eye, as is easily proved, diminishes the acuteness of vision and for this reason apparently the magnification and the perception of depth for the observing eye. Therefore it seems necessary, not to change the conditions for the formation of image in the eye, in applying optical apparatus for subjective use.

5. According to the doctrines of physiological optics, the defective aperture of the rays and the acuteness of vision do not immediately determine the perception of size. But these factors do determine the depth of focus which depends on $\frac{p}{\sigma}$, where p denotes the diameter of the pupil and σ is the angular measurement of the visual acuity.[9] If $\frac{p}{\sigma}$ diminishes, the depth of focus increases, and *vice versa*. The distinctness of the perception of depth is inversely as the optically defined range of distinct vision (depth of focus). On the other hand the sensation of size is con-

[8] A. Gehlhoff, Zs. f. techn. Phys. **66**, p. 477 *et seq.*

[9] Cf. Rohr, Brit. Jour. of Phot. **48**, p. 454, 1901, and S. Czapski, Grundzüge der Theorie der optischen Instrumente, p. 256, 267, 1904.

nected with the accommodation of the eye. Any object seems smaller in proportion as the accommodation is greater or even when the sense of greater accommodation is excited, and this is the case although the image on the retina is unchanged in size. We will assume now, that if anywhere a variation of the range of distinct vision takes place, the accommodation or the innervation of accommodation increases as far as possible without perceiving the image less sharply (*Principle of maximum accommodation*).[10] If the range of distinct vision increases, a point is approached at which the depth begins to be practically infinite and it is, therefore, useful to focus at a nearer point. This does not mean that these focusing impulses are always connected with a real accommodation, for we would then see the nearer point as sharply as the point at which we focused previously. On the contrary, impulses to relax the accommodation take place if the depth of focus decreases again because points at a greater distance are now also distinctly visible and thus the depth of the visual space is enlarged. By this conception, all the experiments mentioned above are intelligible: whether the pupil p or the angular size of visual acuity σ are changed,[11] the innervation to focus begins and together with it a variation of the impression of size and depth.[12]

[10] J. K. Horovitz, Sitz. Akad. Wiss. Wien. vol. 130, 1921; Beiträge zur Theorie des Sehraums.

[11] It is necessary to mention, that a change of the pupil also entails a variation of visual acuity (but not *vice versa*).

[12] It is partly due to the influence of limiting the field of view, that the diminution of the object, when seen through concave spectacles, is much more intensely felt (as shown by Isakovitz), than the diminution of the image on the retina alone could bring about. Therefore, the variation of the depth of focus also must be considered: the entire depth being greater than without spectacles, because $\frac{p}{\sigma}$ which determines the distinctness is only $\frac{pB^2}{\sigma}$ $(B<1)$.—It may be mentioned here, that also other dysmegalopsies, micropsy after the injection of atropin, macropsy after the use of pilocarpin (eserin) are intelligible if we assume the principle of maximum accommodation. In the first case the innervation is unlimited, the apparatus of accommodation being paralysed: hence micropsy. In the other case the external eye is in a spasm of accommodation, the impulses are arrested: hence macropsy.—For these and also other cases of physiological interest see K. Horovitz, *loc. cit.* and further an article appearing in "Pflüger's Archiv" (Grössenwahrnehmung und Sehraumrelief).

6. To show how these changes of our optical sensations are connected with normal sight, we proceed to consider the conception that the space as seen by us, is an optical transformation (in the sense of Maxwell and Abbe) of the physical space.

In this transformation any point of the object-space has a one to one correspondence with a point of the visual space and the lines of sight remain invariant, while the points at infinity are transformed into a plane at finite distance perpendicular to the axis of vision. Mathematically formulated these conditions give, instead of the usual optical transformation

$$x' = \frac{a_1x+a_2y+a_3z+a_4}{ax+by+cz+d}, y' = \frac{b_1x+b_2y+b_3z+b_4}{ax+by+cz+d}, z' = \frac{c_1x+c_2y+c_3z+c_4}{ax+by+cz+d}$$

the equations:

$$x_v = \frac{a_1x}{ax+d}, y_v = \frac{b_2y}{ax+d}, z_v = \frac{c_3z}{ax+d}, \text{ with the condition}^{13}\ b_2 = c_3 = d.$$

These are well known formulae of projective geometry and give analytically the geometry of a relief-perspective for the general case. The origin of the coördinates lies in the first eye-point.[14] If $d = a_1$, the relief is the image of the real space which the *quiescent* eye sees, if the eye is contemplating the point at which it is focused (the point of view is identical with the first eye-point).[15] If the eye is focused on a point, which is nearer than that which is contemplated, then it is necessary for the restoration of the previous conditions to displace forward the eye until the point of view will be so near to the contemplated point, that the latter again is a focused one. But if the eye remains in the place of the first eye point we must say that the point of view is shifted forward only *virtually:* the psychical adjustment of the eye and the innervation corresponds to a smaller distance. Then we have

[13] As postulated by the invariance of the line of sight. It is not necessary for the visual space to be always symmetrical around the x axis, as in the case of astigmatism.

[14] Cf. Burmester, Grundzüge der Reliefperspektive.

[15] The case $d = a$ was developed by H. Witte, proceeding from experimental facts, without connection with the relief-formulae explained above. Physik. Zs. vol. 19, vol. 20, several articles "Über den Sehraum."

$d < a_1$ ($\frac{a_1 - d}{a}$ being the range of the virtual displacement of the point of view) and the objects seem to be smaller and at a greater distance. Thus the conditions of variation of the visual space can be conceived by the rules given for normal vision.

For an exact theory it would be necessary to take into account these transformations only as a first approximation and to find the infinitesimal transformation which takes into account the apparent changes induced by the moving eye and the curvature on the margin of the field of vision.[16]

SUMMARY

The influence of the effective aperture and the acuteness of vision have been dealt with in reference to well-known facts regarding the eye in connection with an optical instrument. It has been shown, that a reduction of the visual acuity entails a diminution of the apparent size. The importance of these facts for some optical observations and measurements is explained. This explanation is based on the assumption that maximum impulses to accommodate are liberated in such a manner that the distinctness of the image is not blurred when the conditions of the formation of the image are altered. The optical transformation of the object-space into the visual space for a quiescent eye is defined by simple postulates of invariance and is proved to be a relief-perspective. Usually the point of view coincides with the first eye-point. The heteromorphies considered above may be constructed as a virtual shifting forward of the point of view.

FIRST PHYSICAL INSTITUTE OF THE UNIVERSITY,
VIENNA, FEBRUARY 15, 1922.

[16] The present writer is pursuing these investigations in connection with Riemannian geometry.—The observations stated here are qualitative and it would be most interesting to obtain some exact investigations on these points, which it was impossible to undertake here.

THE ABSORPTION OF THE EYE FOR ULTRA-VIOLET RADIATION

BY
WINIFRED P. GRAHAM

The eye, as the most delicate sense organ, has always been of great interest, and any investigation adding anything to the sum total of knowledge concerning it, would seem to be worth while. Several investigations have been carried on to determine the relative sensibility of the eye for light of different wave-lengths, but data concerning the limits of the visible spectrum, and the absorption of the tissues of the eye, is conflicting and lacking in information regarding the source and condition of the material. It is, of course, a well-known fact that the eye is sensitive to only a very small portion of the total radiation. The question naturally arises as to whether the limits of our vision are determined by the wave-lengths that are able to affect the retina, or whether the fluids and tissues of the eye actually do not transmit these certain wave-lengths. Nutting says[1] that the retina is most sensitive to radiation in the blue green between wave-lengths .50 and .55μ; that good seeing requires radiation between wave-lengths .41 and .75μ; and if the source is sufficiently intense radiation as far out as wave-length .321 (ultra-violet) or 1.0μ (infra-red) may be perceived. F. W. Edridge-Green states[2] that the limits of the visible spectrum are practically the lines A and H, A wave-length .764μ for the red and H .3968μ for the violet. He also states that the wave-lengths vary with different persons. It may be seen that there is quite a discrepancy between these two writers.

The production of fluorescence in the eye is an important consideration. Wrong conclusions are likely to be drawn as to the limits of the visible spectrum from such experiments as those of Helmholtz and others. When a portion of the spectrum is exposed and produces merely the sensation of light, there is no

[1] Outlines of Applied Optics, p. 120.
[2] The Physiology of Vision, p. 136.

definite proof that the short wave-lengths actually reach the retina, as they might be transformed into long wave-lengths by the tissues of the eye. This would be equivalent to a source very close to the retina. On the other hand if a slit were used with some characteristic shape such as an arrow, for example, if a distinct image were formed with the slit illuminated with ultra-violet, it would seem to prove that we can see by means of ultra-violet light. Nutting's observation is borne out by a member of the Physics Department of this University who claims to have seen the doublet of the mercury spectrum (.3132μ and .3126μ). It might be possible that about the same wave-lengths get through the average eye but that the sensitiveness to the ones near the absorption band vary with the individual.

Dr. Fritz Schanz studied[3] the effect of the ultra-violet light on the eye and the absorption of the different parts. He used a quartz spectral photometer and a Nernst lamp. The corneas of three people of different ages were investigated. He found that the cornea begins to absorb at .360μ, and at .310 μ the rays of the Nernst lamp were absorbed completely. He studied the lenses of three different people of ages 40, and 28 years, and a child's. He states that the lenses begin to absorb in the blue and absorb extensively in the ultra-violet. Using a quartz spectograph he took photographs of the absorption of the cornea and the lens. The latter was pressed between two quartz plates to a thickness of 3 mm. Since the lens in the region of the pupil is considerably thicker than 3 mm, this would only give the partial absorption. His plates show that the cornea begins to absorb at .360μ and absorbs completely at .300μ and the lens absorbs completely at about .350μ on a forty-second exposure. This exposure varied from twenty to forty seconds.

In a second article[4] in 1920, he apparently refutes the data given in the former discussion. This time he used a quartz spectograph. He states that nearly all the ultra-violet is absorbed but although

[3] Über die Veränderungen und Schädegungen der Augen durch die nicht direkt suchtbaren Lichtstrahlen. Archiv. für Ophthalmologie, **86**, p. 549; 1913.

[4] Der Gehalt des Lichts an Ultraviolett. Archiv. für Ophthalmologie **102**. 103 p. 158; 1920.

that which is left reaches the retina, it does not produce the sensation of light but of fluorescence. By using an intense source he says it is possible to perceive light up to .392μ, and if there is light beyond this it is caused by the fluorescence of the retina.

Dr. W. E. Burge gives[5] an absorption spectrum of the cornea of a rabbit which transmits wave-lengths as short as .297μ.

The object of this experiment was to determine what wave-lengths in the ultra-violet region get through the tissues and liquids of the eye, or in other words, what light, in the ultra-violet region actually reaches the retina. It was also determined to compare the absorption of the various parts of the eye and to point out, as much as the material which could be procured would permit, the change in the absorption due to disease, solution in formaldehyde, etc.

The measurements are not as numerous as was wished on account of the difficulty of obtaining material. Human eyes were desired immediately after death and before embalming, or soon after removal in the case of enucleation operations. The specimens procured by the latter method can usually only be used in part because of some pathological condition.

APPARTUS AND MATERIALS

A quartz-prism spectroscope was used, with a dispersion of about 4 inches. A cadmium spark furnished the source of radiation. A transformer connected with nine storage cells gave a good spark across a gap of about a centimeter. The electrodes were sticks of cadmium formed by drawing the molten metal up into small glass tubes. These were stuck through a hollow cork and connected to some Leyden jars and thence to transformer.

A holder was made for the corneas examined out of thin copper sheeting. Two pieces were cut approximately two inches square and an oblong hole cut in each $\frac{1}{2}$ cm$\times 1\frac{1}{2}$ cm. A fold of a half centimeter was then made along the edge of one and the other trimmed off an equal amount. Thus one slipped into the fold and the holes coincided. The cornea to be examined was cut off where it joined the sclerotic coat and placed between the two

[5] "The Production of Cataract." Archives of Opthalmology, 47, p. 12; 1918.

plates, covering the openings. The edges opposite the fold were then placed in a clamp.

Since the crystalline lens in a fresh condition is fairly soft, stiff paper was found very satisfactory as a holder. A cardboard screen was used about 8 by 10 inches with a circular hole of an inch diameter cut in it. The screen was then covered with black paper on each side. A round hole, slightly smaller than the lens to be examined was cut in either side of the black paper, concentric with the hole in the cardboard. This served as a very convenient socket for the lens.

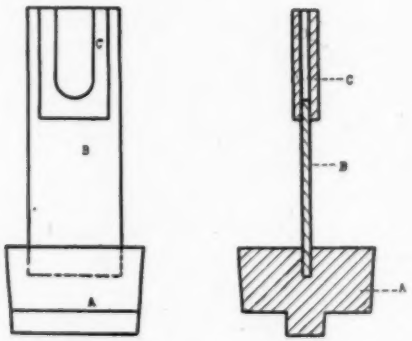


FIG. 1

The liquids of the eyes were put in a small quartz cell for examination. An oblong cut with a semicircular bottom was made in one end of a cover glass, see Fig. 1. Quartz plates of 2½ mm thickness were fastened on either side of this with sealing wax. A small container was thus formed with 1½ mm thickness, 1 cm width, 2½ cm depth. This was very satisfactory as four or five drops could be examined.

The animal eyes used (specimen numbers 1–6) were those of cows, pigs, and sheep. These were mostly secured while they were yet warm, dissected and used the same day. Some were put on ice and dissected and used some four or five days later.

Specimen No. 7 of the human material was secured from an enucleation operation. It was the eye of a man aged twenty

years who had been struck in the eye two years previously with a pen point. The puncture seemed to be completely healed as there was no mark on the cornea which appeared to be normal. The aqueous humor also seemed normal. On dissecting the eye the lens was found to be completely calcified. The choroid had been nearly all absorbed and in its place was a layer of salt. The vitreous humor was very thin and resembled bloody water.

Specimen No. 8 was the lens taken out in a cataract operation. The cataract was mature.

Specimen No. 9 was a lens obtained from a cataract operation. This cataract was not so fully developed. Both these lenses were intact when received and both of the patients had the perception of light and darkness. The lenses appeared opaque in the center but seemed to transmit quite a good deal of light around the edges.

Specimen No. 10 was the lens of an infant of 4 months. The eye had been infected at birth. The lens only was used and it appeared perfectly normal. This view was corroborated by the physician and oculist in charge.

PROCEDURE

The method used in determining the amount of absorption was one of comparison. Plates were taken of the sparks of cadmium, zinc and tin. The wave-lengths on these were marked, see Fig. 2, by comparison with the photographs and data given by Eder and Valenta.[6] Cadmium was found to be most satisfactory for this purpose, as has been mentioned before, because the groupings of the lines are very characteristic and the spark can be easily maintained for a comparatively long time. Spectral photographs were then taken with the material in front of the slit and compared with the original photograph. In some cases the material only covered part of the slit so that a comparison spectra was given on the same plate.

The plates were examined with a small lens and the last line toward the ultra-violet end of the spectrum was taken as the last

[6] Atlas Typischer Spektren, J. M. Eder and E. Valenta.

wave-length to be transmitted, or the beginning of the absorption band.

A cut was made in the edge of the cornea and the aqueous humor was removed. The cornea was then cut off and the lens taken out. Then the posterior of the eye was cut into and a small amount of the clear vitreous removed.

The cornea was put in the holder mentioned above and placed before the slit of the spectroscope. Exposures of varying time were made in order to determine the amount of absorption.

The lens was placed in the cardboard screen and the light from the spark focused by means of it on the slit.

The humors were placed in the quartz cell and put directly in front of the slit and very close to it.

DATA AND RESULTS

The data obtained are given in Tables 1 to 4.

TABLE 1
Cornea

No. of Specimen	Kind	No. of Plate	Time of Exposure	Condition of Specimen	Last Wave-length
1	Cow	13	1½ min.	In formalin	3251
1	"	14	3 "	" "	3251
2	"	20	1 "	Fresh	3134
5	Pig	29	1 "	4 days on ice	3251
7	Human	36	30 seconds	Fresh	3066
7	"	37	45 "	"	2981
7	"	38	60 "	"	3066
7	"	39	1½ min.	"	2981
7	"	40	2 "	"	2981
7	"	41	25 "	"	2981

Seed dry plates number 26 were used except for spectrograms 15 and 16 for which Seed Process plates were employed. The lines were often very faint and at times it was hard to tell just exactly where the last line was located. The last wave-length toward the ultra-violet as given in the last column of the data is only a close approximation since when partial absorption has set in only the strong lines of the source show. Also it might be possible for the transmission to continue for several wave-lengths further and not

TABLE 2
Crystalline Lens

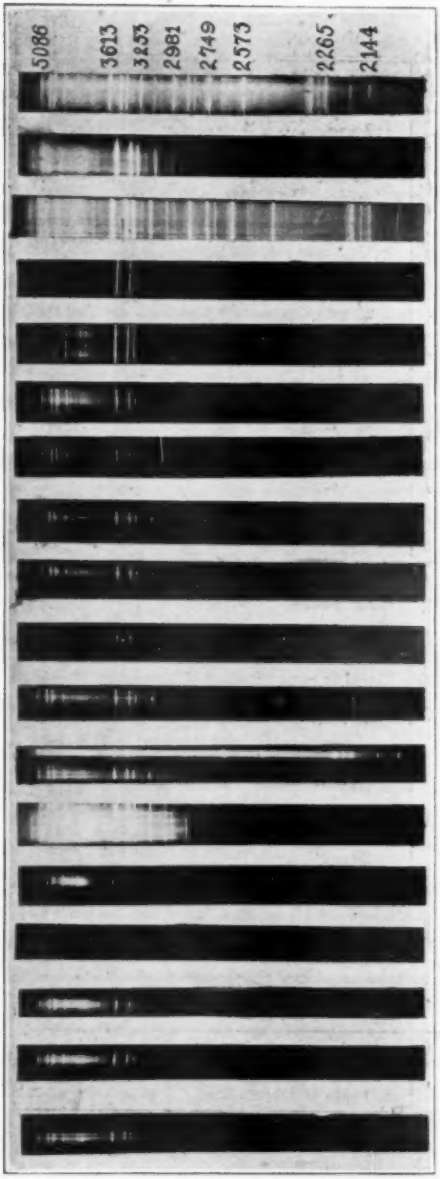
No. of Specimen	Kind	No. of Plate	Time of Exposure	Condition of Specimen	Last Wave-length
1	Cow	15	3 min.	In formalin	3613
1	"	16	3 min.	" "	3613
2	"	19	15 sec.	Fresh	3251
2	"	18	30 sec.	"	3251
3	sheep	21	45 sec.	Kept 3 hours	3251 (very faint)
3	"	22	60 sec.	Fresh	3134
3	"	23	18 min.	"	3134
3	"	24	45 min.	"	3134
10	Human infant	50	1 min.	Fresh	3134
10	"	49	20 "	"	3134
8	Human	44	25 "	"	4415
9	"	45	25 "	"	3404
6	Pig	33	45 sec.	5 days on ice	3251 Not very definite
6	"	34	45 "	"	3134
6	"	35	20 min.	"	3134

TABLE 3
Aqueous Humor

No. of Specimen	Kind	No. of Plate	Time of Exposure	Condition of Specimen	Last Wave-length
4	Sheep	27	15 sec.	Fresh—kept 3 hours	2145 (very faint)
4	"	26	30 "	"	2145
4	"	25	45 "	"	2145
4	"	28	60 "	"	2145
7	Human	42	20 "		2573
7	"	43	40 "		2573

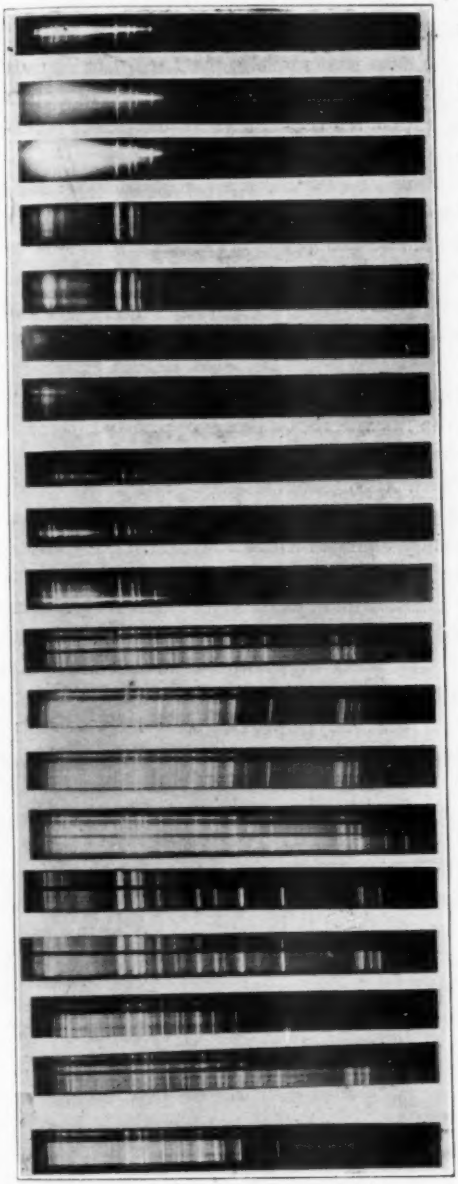
TABLE 4
Vitreous Humor

No. of Specimen	Kind	No. of Plate	Time of Exposure	Condition of Specimen	Last Wave-length
5	Pig	31	15 sec.	4 days on ice	2265
5	"	30	30 "	"	2313
5	"	32	1 min.	"	2265



1. Cadmium
2. Cd. Cover glass
3. Cd. Water
13. Cd. Cornea (cow) 1.5 min. in formalin
14. Cd. Cornea (cow) 3 min. in formalin
20. Cd. Cornea (cow) 1 min. Fresh
29. Cd. Cornea (pig) 1 min. 4 days on ice.
36. Cd. Cornea (human) 30 sec. Fresh
37. Cd. Cornea (human) 45 sec. Fresh
38. Cd. Cornea (human) 60 sec. Fresh
39. Cd. Cornea (human) 1.5 min. Fresh
40. Cd. Cornea (human) 2 min. Fresh
41. Cd. Cornea (human) 25 min. Fresh
15. Cd. Lens (cow) 3 min. in formalin
16. Cd. Lens (cow) 3 min. in formalin
19. Cd. Lens (cow) 15 sec. Fresh
18. Cd. Lens (cow) 30 sec. Fresh
21. Cd. Lens (sheep) 45 sec. kept 3 hours

FIG. 2



22. Cd. Lens (sheep) 1 min. Fresh

23. Cd. Lens (sheep) 18 min. Fresh

24. Cd. Lens (sheep) 45 min. Fresh

50. Cd. Lens (human infant) 1 min. Fresh

49. Cd. Lens (human infant) 20 min. Fresh

44. Cd. Lens (human) 25 min.

45. Cd. Lens (human) 25 min.

33. Cd. Lens (pig) 45 sec. Kept 5 days on ice.

34. Cd. Lens (pig) 45 sec. Kept 5 days on ice.

35. Cd. Lens (pig) 20 min. Kept 5 days on ice.

27. Cd. Aqueous Humor (sheep) 15 sec.

26. Cd. Aqueous Humor (sheep) 30 sec.

25. Cd. Aqueous Humor (sheep) 45 sec.

28. Cd. Aqueous Humor (sheep) 1 min.

42. Cd. Aqueous Humor (human) 20 sec. Fresh

43. Cd. Aqueous Humor (human) 40 sec. Fresh

31. Cd. Vitreous Humor (pig) 15 sec. Kept 4 days on ice.

30. Cd. Vitreous Humor (pig) 30 sec. Kept 4 days on ice.

32. Cd. Vitreous Humor (pig) 1 min. Kept 4 days on ice.

FIG. 3

show on the plate if there was no line in the cadmium spectrum where the absorption began.

It can be seen from the data that most of the absorption bands begin rather abruptly, that is the region of partial absorption is not very great, as long exposures did not bring out many additional lines.

Although the aqueous humor of specimen 7 seemed to be normal, it is apparent from Table 3 that it was not. It seems very probable to suppose that it had absorbed additional salt as well as the posterior part of the eye and the lens. The cornea of specimen 7 (Table 1) seems to have been normal, or at least nearly so.

The following conclusions may be drawn from the foregoing data:

1. The combined tissues of the eye absorb the ultra-violet radiations up the neighborhood of .3134μ.
2. The lens has the largest region of absorption.
3. Formalin changes the absorption.
4. Any injury or disease tending to increase the salt content in the eye radically changes the absorption.

Further, it might seem reasonable to suppose, although the data here given are scarcely definite enough to say conclusively, that the absorption in the animals' eyes does not differ radically from that in the human.

In conclusion, I wish to thank Dr. R. S. Minor for his help; Drs. L. D. and A. S. Green for kindly supplying me with material; and all others who made this work possible.

INSTRUMENT SECTION

A NEW PRINCIPLE AND ITS APPLICATION TO THE LUMMER-BRODHUN PHOTOMETER

BY
E. P. HYDE and F. E. CADY

With the development of different types of photometers from the 18th Century "shadow" form of Rumford through the Ritchie "wedge" and the Leeson-disk improvement of the Bunsen "grease spot," photometric accuracy increased until it reached what appeared to be the maximum in Lummer and Brodhun's contrast type of cube. This was some thirty years ago and the instrument is today quite generally accepted as the one capable of giving the best results where the highest accuracy is desired with the possible exception of those cases involving decided color differences. The secret of the sensitivity lies in the ability of the eye to detect small differences in contrast. While the principle and design of the instrument are well known to those engaged in regular photometric work it will be described for the benefit of others and in order to make the modifications more clear.

The cube, Fig. 1, is made of two triangular glass prisms ABC and ADC cemented together at their hypotenuse faces AC with canada balsam. At Aa, bc, de, the surface of one cube is cut away so that light from the right is reflected in the direction of sight, while light from the left is reflected out of the field of view. As a consequence the observer on looking in the eyepiece of the instrument sees a figure made up of two adjacent semicircles each containing a trapezoid.[1] In the position of balance or when the illuminations on the two sides of the photometer disk are equal, the two trapezoids stand out with equal sharpness from their surrounding backgrounds. If the photometer head is moved and thus thrown out of balance one of the trapezoids will tend to get darker and the other will tend to approach in brightness that of the surrounding background. A movement of the photometer

[1] Instruments made in this country use a segment of a circle in the place of the trapezoid.

head in the opposite direction will reverse this condition and make the other trapezoid less distinct. In the position sought for, the contrast between each trapezoid and the surrounding background is the same. This contrast results from the presence at *EA* and *FD* of two plain pieces of clear glass which by reason of surface reflection decrease the transmitted light by about 8 per cent.

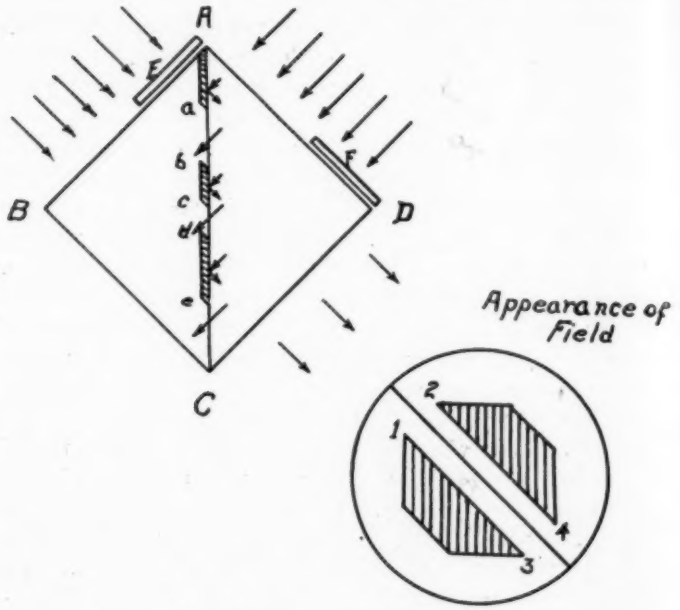


FIG. 1. Schematic diagram of Lummer and Brodhun Photometric Cube, Contrast Type.

These pieces may be removed if it is desired to dispense with the contrast principle and use the cube for a simple equality-of-brightness or match photometer.

It was while studying the Brace spectrophotometer that there occurred to one of the authors the possibility of increasing the accuracy of photometric settings by the introduction of a new principle. It may be recalled that in one form of the Brace spectrophotometer the photometric field is made up of three parts,

Fig. 2, the center receiving light from one source while the upper and lower portions receive it from the other source. In the condition of balance the field "*B*," which receives its light from the source *S* is of the same apparent brightness as the two fields "*A*" and "*C*" receiving light from the source *R* and which are of the same actual brightness if the instrument is in adjustment. If it is assumed that the eye can just detect a difference in brightness of 1% in a field of view of this kind, it is evident that there would be an apparent balance for all values of brightness of field "*B*" lying within ± 1% of the brightness of "*A*" and "*C*." That is, the range of a setting would be ±1%, or, in total, 2%.

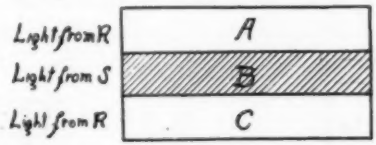


FIG. 2. Field of view in Brace Spectrophotometer

Suppose, now, that by some suitable optical means the brightness of field "*C*" were made definitely 1% less than the brightness of field "*A*." In a condition of balance the brightness of "*B*" obviously would now lie between the brightness of "*A*" and the brightness of "*C*" which has been made 1% less than the brightness of "*A*." For when "*B*" is in actual match with "*A*" it is, by hypothesis, 1% brighter than "*C*," and the difference in brightness between "*B*" and "*C*" would be detected. Similarly, where "*B*" is in actual match with the modified "*C*" it is 1% less bright than "*A*," and again the difference would be detected. Hence, with this modified field the range of error would be only 1% instead of 2% as in the original arrangement.

It is true that the center of balance is shifted from the brightness of "*A*" to a value ½% less, but since for other obvious reasons the substitution method of measurement would always be employed this shift of ½% offers no difficulty. It should be noted that this principle was subsequently used and described by

Pfund[2] in an article on a new photometer involving the use of mirrors.

Although this principle was thought of in connection with the Brace spectrophotometer it is of general application, and was first actually tried out in connection with the Lummer-Brodhun contrast photometer. Since with this instrument the condition of balance consists not in brightness equality but in equal brightness contrasts the application of the principle involves the establishment of unequal contrasts in different parts of the field. Thus if the contrast of the field 1–3 (Fig. 1) is made to decrease from 1 to 3, and that of the field 2–4 to increase from 2 to 4, by some chosen small amount, a resultant total field of view is obtained such that the range of balance is greatly reduced, as is apparent by a similar process of reasoning to that presented in the first case.

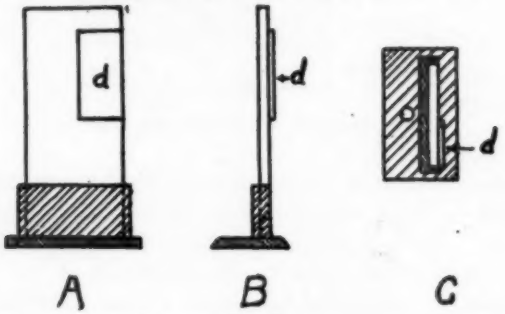


FIG. 3

A, B, C front, side and top view of enlarged strips showing attachment of small absorption piece, *d*

The practical difficulty of applying this principle to obtain a graduated contrast lies in the fact that the glass strips "*E*" and "*F*" absorb by reflection and any variation in thickness produces only negligible changes in the absorption and therefore in the contrast. This difficulty was overcome by enlarging the clear glass strips to cover the entire faces of the cube, and then attaching to these by canada balsam other strips of the same size as the original strips "*E*" and "*F*," but made of smoked glass, which would absorb in proportion to their thickness, see Fig. 3. By

[2] *Phys. Review 4*, p. 477, 1914.

making these strips wedge-shaped, a graded absorption and therefore a graded contrast could be obtained.

Moreover, by accomplishing the contrast in this way it is possible to make the average absorption, and, therefore, the average contrast any amount that might be desired, instead of having it determined by the reflection of clear glass strips which give a fixed absorption of about 8%. This led to another increase in sensibility. Lummer and Brodhun had found by experiment that 3½% contrast gave the most sensitive photometer, but they apparently thought of no easy way of accomplishing this, and satisfied themselves with the 8% contrast obtained by the clear glass strips. With the introduction of the scheme described above for getting graded contrast it was a simple matter to choose a density and thickness of smoked glass that would yield an average absorption of 3½% with the added feature of a properly chosen graded contrast.

Messrs. Franz Schmidt and Haensch of Berlin undertook to construct for us a photometer involving these two features, and delivered the instrument to us over ten years ago. Just what gradation in contrast was aimed at the authors have now forgotten. At that time some measurements were made using a photometric method to determine whether any gradation was present, but none was found. However, recent measurements using a much more sensitive method gave a difference in one direction of 0.7 per cent in one piece and in the opposite direction of 0.2 per cent in the other piece indicating an effort to have them comply with the principle. At the time of the latter measurements, readings were taken on six other pairs of low contrast strips, but in all these cases whatever variation was found was in the same direction from top to bottom or bottom to top on both strips and so would not show the application of the principle referred to. The absorption of these strips was such as to give a contrast of about 5 per cent. In spite of the lack of opposed variation in absorption, the instruments using these strips have shown a very much greater sensitivity than the ordinary type, doubtless due to the decreased contrast.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
APRIL, 1922.

AN ELECTROMAGNETIC METHOD OF DETECTING MINUTE IRREGULARITIES IN CURVATURE OF SPHERES AND CYLINDERS AND OF CONTROLLING THE OSCILLATIONS OF A MASS OF METAL SUSPENDED BY MEANS OF A TORSION FIBRE

BY
ALEXANDER MARCUS

It is well known that a single unidirectional alternating magnetic field is equivalent to two constant fields revolving with equal angular velocities in opposite directions and it is commonly believed that a coil of wire or a mass of metal placed in a uniform alternating field so that it can rotate will remain at rest if it happens to be at rest. The reason for this is that the two oppositely revolving fields which may be considered equivalent to the single impressed alternating field, induce two[1] symmetrical polyphase systems of currents of the same frequency and amplitude. Therefore, the mechanical reactions between these currents and their corresponding fields are equal and opposite and the body experiences no resultant torque. If the body be given a start in either direction the opposite torques will no longer balance. For, suppose the body be given an angular velocity ω, and the angular velocities of the fields are p, and $-p$ respectively, then relative to the body one of the fields

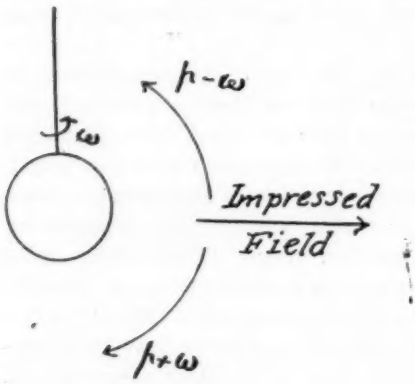


FIG. 1

[1] Steinmetz: A. C. Phenomena, 4th Ed., pp. 639–642.

will revolve with angular velocity p-ω and the other with angular velocity $p+\omega$. The two systems of induced currents now have frequencies proportional to $p-\omega$ and $p+\omega$ respectively and have different amplitudes. Therefore the mechanical reactions between the currents and the fields inducing them will no longer be equal and opposite. This is in essence the theory of the single-phase induction motor and explains why such a motor is not self-starting.

However, the writer has recently found in the course of an experiment with a copper ball suspended by means of a torsion fibre perpendicular to a uniform alternating magnetic field, that the ball will start to rotate even if it be perfectly stationary when the field is thrown on. It seemed to act like a self-starting induction motor, and inasmuch as according to well established electrodynamic theory such a motor cannot be self-starting, it became a matter of importance from theoretical as well as practical considerations to explain the existence of the extra torque. It was apparent that the starting torque was not an effect of the free electromagnetic oscillations produced by the sudden application of the field, for, as long as steady conditions in the field were maintained, the ball continued to oscillate about a definite new zero position. On repeating the experiment with both solid and hollow spheres and cylinders of different non-magnetic materials, it was found that the extra torque varied directly as the intensity of the "skin effect" for the different samples. Owing to the fact that all the spheres and cylinders had highly uniform curvature, it seemed difficult to attribute the effect to irregularities in curvature. Nevertheless this was assumed as a working hypothesis and two identical hollow circular cylinders were suspended alternately in the field. The observed deflections from the normal zero position were nearly equal for the two samples. When one of the samples was compressed so as to have an approximately elliptic cross-section, with the long axis about twice as long as the short one it gave a deflection many times larger than before distortion. Furthermore, the torque was always in such a direction as to tend to bring the long axis into coincidence with that of the field. A bronze ball-bearing was then suspended in the field and no starting torque was detected.

On linking together the two facts, first, that the torque on the asymmetric body increased with the "skin effect" and, second, that the torque always tended to make the body set its long axis in the direction of the uniform field the writer came to the conclusion that his discovery was only a new illustration of a fact already established but not generally known, namely:—

A conductor placed in a variable magnetic field tends to behave like a diamagnetic substance both in the way it disturbs the field and in the way it tends to move under the influence of that field.[2] The reason for this is that as the frequency of the impressed field rises the induced currents and magnetic fields tend to become concentrated in a thin layer near the surface and consequently the space within the body is traversed by a weaker field than before the introduction of the conductor. The higher the frequency the weaker will be the internal field. Indeed even in the case of iron the penetration of the field and currents may be so small at high frequency that the total flux through the space occupied by the body may be less than what it would be, were the iron removed. Dr. Louis Cohen[3] has proved that in an iron cylinder of one centimeter radius subjected to an alternating magnetizing force having a frequency of a million cycles per second the total flux through the iron is only seven-tenths as much as the flux through that space before the introduction of the iron. It should also be remembered that whenever any body of a given permeability is immersed in a medium of a different permeability it will tend to move toward the region of lowest potential.[4] For this reason a diamagnetic substance in a uniform magnetic field tends to turn its longest dimension in the direction of the field in contradiction to the statement frequently made that a diamagnetic substance turns its longest dimension across the field.

This diamagnetic behavior of a conductor in an alternating field has two practical applications. First, it affords a very sensitive method of detecting irregularities in curvature in metal spheres and cylinders and secondly, may serve as a means of

[2] J. J. Thomson: Recent Researches in Electricity & Magnetism, pp. 556–7.
[3] Calculation of Alternating Current Problems, p. 38.
[4] Poynting & Thomson: Electricity & Magnetism, pp. 257–259.

starting, stopping, and of controlling the oscillations of a mass of metal in experiments involving torsional oscillations of such a mass. In both of these applications a pair of Helmholtz coils will be found convenient for the production of a uniform field. The ball should be suspended midway between the coils. The mean distance between the centers of the latter should be about equal to the radius of the coils. If it be desired to test the ball for irregularities in curvature suspend it like a galvanometer coil with a small mirror attached to the suspension fibre for the purpose of observing the deflections of the ball. Before applying the field let the ball come to perfect rest and note the position on a scale of the reflected beam of light. Throw on a field of ordinary frequency and determine the new zero about which the oscillations occur. The sensitivity of the method depends upon the magnitude of the "skin effect" for different materials and hence varies directly as the square root of the frequency of the impressed field and inversely as the square root of the resistivity of the material. In this case as with galvanometers the sensitivity depends also upon the stiffness of the suspension fibre and upon the distance of the scale from the mirror. Since the torque on the ball depends upon the reaction between the impressed field and the currents induced by that field, it will vary directly as the square of the amplitude of the field.

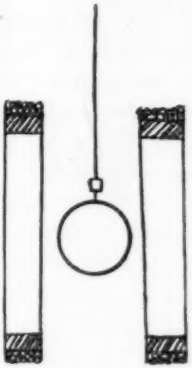


FIG. 2

The writer had a mass of pure copper cast into the shape of a ball about two inches in diameter. It was then turned down and polished to be as nearly spherical as possible. Measurements with a vernier caliper showed deviations of any single diameter measurement from the mean, of about one-half of one per cent. On suspending the ball in a field alternating with a frequency of sixty cycles per second and having an effective intensity of about thirty gausses the torque due to the diamagnetic behavior of the

ball gave a deflection of several centimeters on a scale about a meter from the mirror on the suspension.

The second practical application of the diamagnetic behavior of a conductor in an alternating field is to the control of the oscillations of a torsion pendulum. There are some electrodynamic experiments and others of a purely mechanical nature based on the use of the torsion pendulum and require that the motion shall be purely torsional. For example, from the decrement of the oscillations of a mass of metal vibrating as a torsion pendulum in a constant field, it is easy to compute the resistivity of the material. Or one may wish to determine the coefficients of viscosity and of rigidity of a substance in the form of a fibre by the method of the torsion pendulum. In such experiments, mechanical methods of starting the oscillations are quite apt to produce a variety of undesirable modes of vibration. Moreover, in order to avoid troubles due to tremors of the building, it is often necessary to mount the apparatus in places not readily accessible. The magnetic control of the oscillations makes it possible to start the motion and to establish any amplitude by the mere closing of a switch. The only extra apparatus required is a pair of field coils, the metal being placed midway between them. The increase or decrease in amplitude is produced by closing the switch when the motion is either in the same direction as the torque due to the field or in the opposite direction.

The writer wishes to acknowledge his indebtedness to Mr. Robert Dressler, Mechanician of the Department of Physics of the College of the City of New York for his valuable assistance in winding the field coils and in preparing some samples of metal for the experiments.

COLLEGE OF THE CITY OF NEW YORK

AN IMPROVED FORM OF NICHOLS RADIOMETER

BY
B. J. SPENCE

Abstract. The improvement over the customary forms of the Nichols radiometer consists in the reduction of the moment of inertia of the rotating system to such a value that the system has a period of eight seconds and a sensitivity equal to that of the thermopile or bolometer under their best working conditions. A comparison of the thermopile and the radiometer is given.

Numerous articles have been written dealing with the descriptions and relative merits of the radiometer, thermopile, and bolometer. Investigators who have used these instruments are familiar with the difficulties in the way of zero drift and fluctuations when the bolometer or thermopile is used in conjunction with the sensitive Thomson astatic galvanometer. The system of the astatic galvanometer is of small mass and moment of inertia. It is suspended by a fine quartz fibre in a relatively strong non uniform control field. Slight building vibrations or tremors cause a displacement of the system to other positions in the field of different intensity and directions giving rise to an amplification of these disturbances.

In the course of a study of some infra-red absorption spectra, using a grating of relatively large dispersion and resolving power and small energy of the radiating source, it was found necessary to abandon the thermopile and bolometer and attempt to develop a radiometer of the Nichols type which was free from the usual objection of the long period possessed by such an instrument.

The Nichols[1] radiometer as customarily constructed consists of a pair of mica vanes of comparatively large dimensions and mass fastened by fine glass rods to a glass staff with a mirror attached for purposes of observation. The system is suspended by a fine quartz fibre in a container pumped out to a pressure of approximately .02 mm Hg. Such a system has a large moment of inertia and when suspended by a sufficiently fine fibre to give a sensitivity of the order of magnitude similar to that of the bolometer,

[1] Ann. der Physik, *60*, p. 402; 1897.

the period in some cases is 60 seconds or more. This constitutes the main objection to its use in spite of the remarkable stability and freedom from zero drift.

Accordingly, it seemed feasible to attempt to reduce the moment of inertia of the system and thus its period. To this end two strips of phosphor bronze ribbon .13 mm wide, .018 mm thick and 15 mm long were used for vanes. These strips were laid parallel about 4 mm apart and between them a finely drawn glass staff 40 mm long. The strips and glass staff were held together by finely drawn glass cross pieces by means of the smallest amount of shellac. To one end, which we shall designate as the tower end, was fastened a plane mirror of about 1 mm^2. One face of each strip was blackened with lamp black and alcohol containing a trace of shellac to cause the lamp black to adhere to the strips. The system was suspended from its upper end by means of a quartz fibre in a thick walled iron chamber cylindrical in shape and then pumped out to a pressure of approximately .02 mm Hg. The iron chamber, Fig. 1, mounted on a base with leveling screws had a bore of 25 mm, with 10 mm wall thickness and a length of 20 cm. Near the bottom and opposite each other were cut two windows. The one covered with quartz or rock salt was 3 mm wide and 20 mm long, the other for observation purposes and covered with glass was 5 mm wide and 35 mm long. The top of the chamber was provided with a ground iron plug and mercury seal. The plug served to suspend the system and by rotation to adjust the zero of the instrument. Near the top projected a tube carrying a carefully ground glass stop cock for evacuation purposes. The thick walled iron chamber was used to smooth out possible temperature fluctuations. As an extra precaution against temperature fluctuations the chamber was covered with a layer of felt 10 mm thick. The system thus protected was free from zero drift. It could be operated for a period of hours without a drift of more than 1 mm with a scale at 2.5 meters distant. In addition, it was free from fluctuations due to building disturbances.

It is difficult to make a rigorous comparison of the sensitivity of the radiometer with the bolometer or thermopile. It is cus-

tomary to state the sensitivity of those instruments in terms of the deflection produced by a candle at a meter distance and scale 1 meter distant. Incorrect conclusions may be drawn from such a rating. It does not indicate clearly the working sensitivity. Drift and fluctuations are not usually considered. The thing most desired in infra-red spectroscopy along with sensitivity is reliability of deflection.

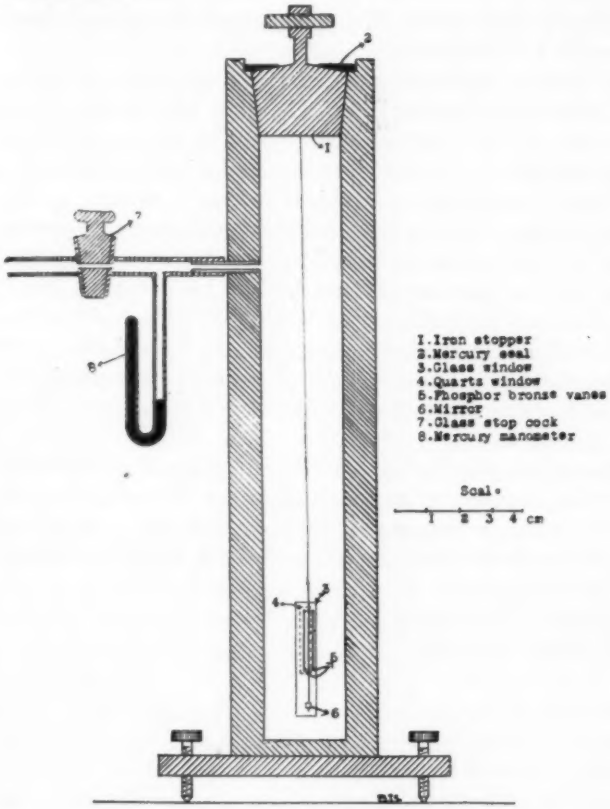


FIG. 1

A comparison was made of a bismuth-antimony bismuth-tin thermopile of 20 receiving junctions and 6 mm² receiving surface,

in series with a low resistance astatic galvanometer of 3 x 10^{-10} amperes sensibility, with a salt window radiometer of 2 mm^2 receiving surface and a period of six seconds. The comparison was made by subjecting the instruments in turn to the region of the spectrum near the sodium lines. The spectrum was produced by passing the radiation from a Nernst glower through an infra-red spectrometer. Deflections of approximately 50 mm were obtained in both cases. The radiometer deflection was free from zero drift and fluctuation.

In studying transmission spectra, it is customary to return to the zero reading after each deflection but with no zero drift this procedure is not necessary and the objection to the period of eight seconds as compared to a period of 1.5 seconds for the thermopile galvanometer combination is not serious.

In conclusion it may be stated that a radiometer of sensitivity equal to that of the thermopile or bolometer and a period of eight seconds has been developed. The instrument is free from zero drift and fluctuations that would ordinarily prohibit the use of the bolometer or thermopile. It is recognized that such an instrument has not been developed to obtain quantitative measurement of the small amounts of energy such as may be done with the bolometer.

The radiometer was developed in the course of an investigation for which funds were appropriated by the Rumford Committee of the American Academy of Arts and Sciences. Acknowledgment is hereby made of the courtesy of the Rumford Committee for the appropriation.

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A PORTABLE SEISMOMETER

BY
P. G. NUTTING

Vibrations in buildings due to nearby machinery or traffic constitute today a serious engineering problem. Troublesome noises are usually readily traceable to their sources through their pitch. Jars and vibrations are far more difficult to trace. Laboratory forms of seismographs, used in recording earthquake tremors, are of little service because of lack of portability. Stethoscopes and other forms of delicate sound detectors are ineffective in observing vibrations of slow period. The portable seismometer here described was designed for engineering purposes. It weighs but a few pounds and can safely withstand rather rough handling, yet is of ample sensibility for detecting and measuring all ordinary tremors.

The amplitudes to be measured lie in the range from 0.001 mm to 0.1 mm, the former being just perceptible to the hand, the latter being decidedly annoying. Preliminary determinations of amplitudes were made with a very simple form of seismometer assembled in a few minutes from two paper weights, two pieces of drill rod, a galvanometer mirror and a pocket flash lamp. Two strips of plate glass about 2 x 6 inches would serve quite as well as the two paper weights. The lower plate is placed upon the table or desk whose vibration is to be observed. Upon this plate are laid the two pieces of drill rod, 2 or 3 inches long, parallel with each other; then the upper plate placed face down on these as shown in Fig. 1.

A half inch galvanometer mirror is attached to the projecting end of one piece of rod, throwing on a wall the image of the filament of a flash lamp. The two rollers of drill rod or wire must of course be straight, round, and free from burrs. About 1 mm is a suitable diameter. Soft wire may be used if carefully rolled straight. Some drill rod requires grinding to give it a sufficiently circular section.

In operation the upper plate remains stationary while the lower moves with its support. The angular displacement of the reflected light beam is of course twice that of the roller carrying the mirror. Hence the relative linear displacement of light and test plate is in the same ratio as distance of light spot to radius of roller. A vibration of 0.005 mm (easily perceptible to the hand) will give an angular displacement of 0.01 to a roller of 0.5 mm radius and therefore will cause a displacement of 0.2 foot in a spot of light reflected 10 feet away. With rollers 0.2 mm in diameter the sensibility is 5 times as great. Frequencies at least as high as 100 per second are readily observed. Three or four components in a vibration may easily be detected.

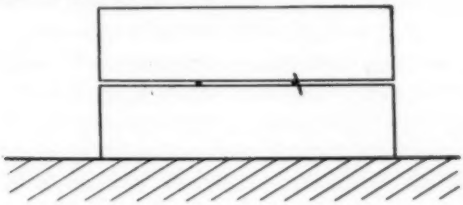

FIG. 1

Such a simple form of seismometer as that just described, although quite serviceable in horizontal positions, is quite useless for determining vertical components. The form of portable instrument finally developed is of very different design although essentially the same in principle (Fig. 2). A mass of lead supported by a diaphragm forms the stationary member while the case moves with the object with which it is in contact. Both rollers project and carry mirrors reflecting light from a miniature flash lamp upon an attached scale.

Fig. 2 shows the construction of the instrument in sectional diagram. The base is a hollow circular box 6 inches in diameter and 1 inch thick of light stiff material such as bakelite micarta. The interior is ½ by 5 inches diameter. Between the halves of this box is clamped a circular diaphragm of similar material about 1/32 inch in thickness, just sufficient to support the lead weight in zero position yet permit the housing to vibrate independently.

The lead inertia weight, 20 x 35 mm in section and 6 inches long, is attached at one end by a thin nut to the center of this diaphragm by means of a brass rod passing longitudinally through it. At the head of the lead weight is a brass block of the same section milled out on either side to carry the glass plates forming the bearing surfaces for the two rollers. On either side of the lead weight are heavy L shaped uprights of heavy strap brass forming the stem of the seismometer. These are firmly screwed to the top of the circular base and are also recessed near the top for bearing surfaces of plate glass at the top; each is screwed to a milled top plate of

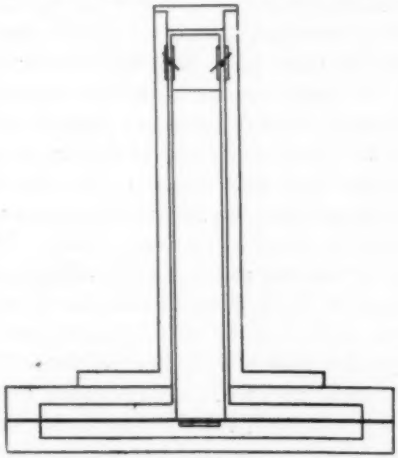

FIG. 2

heavy brass. An axial screw (not shown) through this top plate bearing against the top of the inertia bar serves to clamp its vibrations when the instrument is not in use.

The plate glass bearing surfaces must of course be set accurately parallel but if mounted in a fusible cement such as de Khotinsky's, their setting (before assembly) presents no great difficulty. Final adjustment of separation to the rollers is made by means of shims under the top ends of the brass side plates. When the adjustment is correct the rollers can just be slid into position with the fingers and will remain in place. The rollers may be quickly ad-

justed to parallel horizontal position by means of a steel scale or try square.

Hard steel rod such as drill rod is preferable for this instrument. Such stock is usually not quite round and must be ground between hard flat surfaces in a fixed position. A pair of fine carborundum stones clamped together with a wedge shaped opening between serves very well. A rod diameter of about 1 mm is preferable. The mirrors used are carefully selected half inch plane galvanometer mirrors attached by means of soft wax (to avoid distortion) to a thin light metal plate which is in turn cemented to the roller with fusible cement.

The optical system (not shown in Fig. 2) is simple and readily attached to the top plate. The illuminator is a small pocket flash lamp having the bulls eye lens replaced by a corrected lens forming an image of the lamp filament about 6 inches distant on a printed scale attached at the rear of the top of the instrument. The upward beam after reflection from the second roller mirror is reflected horizontally across the top of the instrument by means of a fixed mirror mounted at an angle of 45°. The beam being twice reflected by rotating mirrors, the sensibility of the portable type of seismometer is of course double that of the simple form shown in Fig. 1.

This instrument registers deflections of from 2 to 40 mm for vibrations ordinarily met with. It is readily portable and has been extensively used by laymen without being damaged. Used on a vertical wall (held in the hand by the stem) it reads as well as on a horizontal table surface, the shift of zero position being of no consequence in reading amplitudes. A simple form of photographic registering attachment has been designed, but this and other refinements have been found superfluous in ordinary engineering observations.

This instrument was designed at the request of the Westinghouse Electric and Manufacturing Co., East Pittsburgh, for their own use and was developed in the Research Laboratory of that company during the winter of 1920–21.

SCHENECTADY, N. Y.

AN INSTRUMENT FOR THE GAMMA RAY MEASUREMENT OF THE RADIUM CONTENT OF WEAKLY ACTIVE MATERIALS

BY
N. ERNEST DORSEY

For the determination of the radium content of a material of unknown composition the emanation method is the only one that can be considered as thoroughly satisfactory. This method, however, requires careful preliminary chemical treatment of the material, and consequently, is quite time consuming. On the contrary, a gamma ray measurement requires no chemical treatment whatever, is expeditious, and when satisfactory precision can be obtained, answers every purpose in all those cases in which it is known from other evidence that the material contains no radio-active material other than those belonging to the uranium-radium family.

The main difficulties encountered in the application of the gamma ray method to weakly active materials are (1) the securing of satisfactory sensitivity, (2) the determination of the correction for the volume distribution of the material, and (3) the determination of the correction for the absorption of the radiation by the material itself. If unlimited amounts of material are available, the first of these difficulties is not serious, and several modus operandi may be devised that will permit of a satisfactory determination of the two corrections mentioned. But the necessity of handling large amounts of material is in itself a disadvantage, and frequently only a small amount of material is available.

It was to meet these conditions that Walter Bothe[1] devised the double cylinder apparatus shown in Fig. 1. The material under test is placed in a thin walled test tube 2.6 cm in diameter, the latter is hermetically sealed and is placed in the cylinder B. The test tube is in all cases filled to a height of 12 cm. The cylinder B is of 3 mm brass covered externally with lead ½ mm thick.

[1] Physik, Zeits., 16 p. 33–36, 1915.

It is 2.8 cm in internal diameter and is 16 cm long; it is carried by the leaf support, and the whole is insulated by the sulphur ball, S. In the figure, the radium standard is shown suspended in the center of B. The correction for the volume distribution is determined by a direct survey, a small tube of radium—essentially a point source of radiation—being placed successively at different points in B, and the corresponding rates of drift being determined. By a similar survey in which the radium tube is

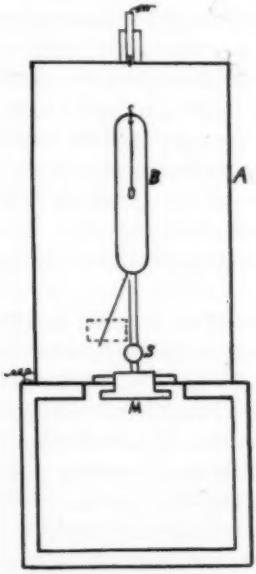

FIG. 1. Bothe's electroscope

buried at various points in fine sand contained in a test tube placed in B, the ratio of the resultant absorption of the radiation by the specimen to the absorption corresponding to the central point of B was determined. This, combined with the observed absorption by the material under test of the radiation from the small tube of radium buried in it so as to lie at the center of B, gives the correction for the absorption of the radiation by the material itself.

For an instrument of this type, Bothe's instrument can scarcely be improved upon from an electrical standpoint, and in the hands of a skilled experimenter should give excellent results. But it has two annoying disadvantages that make it unsuitable for routine work. (1) The opening of the electroscope for every change in the specimen, with its attendant disturbance of the leaf, change in the air in the electroscope, and changes in the convection currents arising from slight changes in the temperature of *B*, keeps the observer doubtful of the confidence that can be placed in the results, and necessitates numerous repetitions. (2) Any slight escape of emanation from the specimen, either from an imperfect sealing of the tube or from a soiling of its exterior with the material, will produce a marked effect upon the observed rate of drift. Consequently in the determination of the correction for absorption, the tube has to be carefully sealed and cleaned at every step. Furthermore, throughout these manipulations care must be taken to prevent the introduction of radium emanation into the air of the room containing the instrument.

It was to obviate these disadvantages that the instrument shown schematically in Fig. 2 was devised. It may be described as a triple cylinder, or reentrant, ionization chamber attached to an electroscope. As in Bothe's instrument, the material under test is sealed in a test tube and placed in the cylinder *B*. The specimen can be introduced and removed readily without disturbing in any way either the insulated system or the air contained in the instrument. If the joints of the instrument are reasonably tight, a small amount of emanation in the air of the room will produce but a minimum effect, and this effect will change in a fairly regular manner and can be readily eliminated from the observations. Consequently, if the material is first aerated in another room, the tube containing it may remain open during the determination of the correction for the absorption of the radiation by the material itself. This greatly facilitates the work. If the instrument were made strictly air tight, it would be unaffected by the presence of emanation in the air, and all the manipulations of the material could be carried out in the same room. These advantages have been secured at the expense of the sensitivity, the electrostatic

capacity of this instrument being much greater than that of Bothe's. But experiments with a tentative instrument of this type show that even with its reduced sensitivity it has a wide field of practical use.

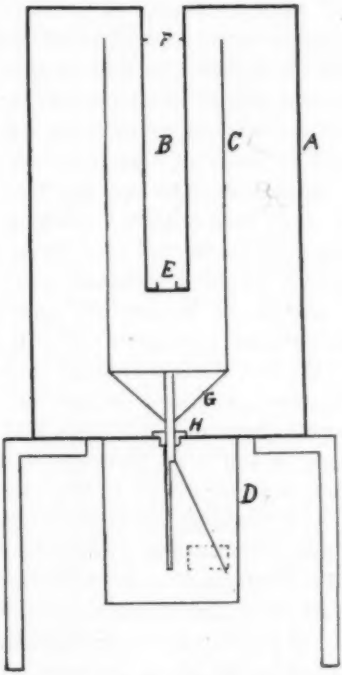

FIG. 2. Triple cylinder electroscope

In the tentative instrument, the cylinder A (Fig. 2) is 33 cm long and 21 cm in diameter; it is lined with lead 3 mm (⅛ in.) thick; lead of the same thickness is used in the construction of B, which is 21.5 cm long and 3.2 cm (1¼ inch) in internal diameter. *C* is a cylinder of aluminum 0.3 mm thick, it is 25.4 cm long, 9 cm in diameter, and is open at both ends; it is supported below by a spider which is carried by a brass rod passing through and supported by the sulphur casting, *H*. Guy wires, *G*, give the spider

the necessary stiffness. The electroscope case, D, is of thin metal and is covered with felt; a heavy lead covered case would be preferable. The insulated system is charged by means of an insulated bell crank switch passing through the walls of D and not shown in the figure. In the bottom of B and coaxial with it is a pasteboard cylinder E about a centimeter long, and near the top of B is a pasteboard ring, F, 1 inch in internal diameter. These serve to center the tube of material and to lift it slightly above the bottom of B. The lead walls of B exert an excessive absorption upon the radiation from material situated extremely near them; consequently it is undesirable for the test tube containing the material to fit B snugly or to rest upon its bottom.

Corrections for the volume distribution of the material and for the absorption of the radiation by the material itself were determined in the general manner described by Bothe, except that the absorption surveys were made in the materials under test. As was to be expected, the transverse surveys show that while the curve showing the relative change in the effective absorption as the radium tube is moved along a radius is essentially the same for all sections throughout a relatively long central section of the column, it becomes much flatter as the ends of the column are approached. For these surveys, a small glass tube containing about 0.1 mg of radium was used.

The sensitivity of the instrument as used was about 2.8 divisions per second per milligram of radium. Specimens of radium concentrates (crude sulphates and carbonates) containing from 7 to 60 micrograms and small preparations containing as much as 300 micrograms of radium have been satisfactorily measured with it to a precision of at least one percent; and with considerable difficulty, duplicate specimens containing only 0.7 microgram were measured with a concordance of 2% and with a departure from the results of an emanation measurement of less than 10%. In one case, 4 samples of the same material dried to different extents were measured. One sample consisted of the material as received and contained over 50% of moisture; two contained about 7%; and one was dry. The actual radium contents of the specimens varied from 9 to 13 micrograms. The four determina-

tions thus made of the radium content per kilo of the dry material covered an extreme range of only 0.7%.

Selected one-inch test tubes were used and these were always filled to a height of 6 inches; this amount of material weighed between 40 and 110 g, depending upon its nature. After the desired amount of material had been placed in a tube, a thin cork disc to which was attached a wire hook was pushed down firmly against the material and was covered with a thick layer of melted sealing wax. After the tube has been sealed, it may be put aside until the radiation has reached its maximum value before a gamma ray measurement is made, or the maximum value of the radiation may be determined, by the well known method of extrapolation, from a series of measurements made at suitable intervals. In the latter case, the graphical method[2] described by the author some years ago is very convenient.

Assuming that 70 g. of material is used, a 5 microgram specimen will correspond to material containing about 7×10^{-8} gm Ra per g of material. The tentative instrument can therefore be satisfactorily used for materials containing between 7×10^{-8} and 4×10^{-6} g Ra per g, and can, though with difficulty, be used with a precision of at least 10% for material containing only 7×10^{-9} g Ra per g. The last is a little less than the radium content of ore containing 3% U_3O_8.

Though the instrument is well suited to the determination of the radium content of concentrates, it is not suitable for work with low grade ores. For such work a more sensitive instrument, or one utilizing a larger amount of material is required.

WASHINGTON, D. C.

[2] Phys. Rev., (2), 14, p. 173, 1919.

THE EFFECT OF A PHOTO-ELECTRIC MATERIAL ON THE THERMO-ELECTRIC CURRENT IN HIGH VACUUM AUDION BULBS

BY
THEODORE W. CASE

The author in his work on barium and strontium photo-electric reactions, which were first noted in Audion bulbs of oxide-coated filament type,[1] has observed peculiar effects of these light reactive substances on the thermo-electric current when the active coating is somewhere between the plate and filament. The active material can be used as a light reactive grid and made to control the thermo-electric current flowing between filament and plate. The outstanding feature of placing an electro-positive photo-electric material near the plate is to very greatly reduce the thermo-electric current to the plate when the photo-electric material is in comparative darkness.

In this latter condition it requires considerable voltage to get any thermo-electric current to the plate. If light is allowed to fall on the photo-electric material, the effect is to instantly allow the thermo-electric current to pass to the plate, and consequently large changes in the thermo-electric current may be easily induced by moderate changes in light intensity. The observed increase of thermo-electric current for low light intensities is not of the trigger action variety, but the maximum current obtainable is limited by the possible thermionic current which would obtain if the light active substance were not present in the bulb; therefore the current obtainable cannot be directly proportional to all light intensities.

It is quite easy to construct cells of this type by including an oxide-coated filament opposite a plate in a vacuum bottle and introducing potassium distilled from a glass side tube upon a grid, or upon the glass wall of the bulb if it be made tubular between the filament and plate. In operation, the voltage between the

[1] Paper before A. E. S., April 21, 1921.

heated filament and plate (which latter is made positive) should be so adjusted that very little thermo-electric current flows when no light is on the potassium. Upon illuminating the potassium the increase of the thermo-electric current will be observed.

The author believes that this type of action warrants the trial of many substances, either conductors or non-conductors near the plate in the form of a grid, which may be thus studied for reaction to different types of radiation, including X-rays, with possibly very interesting results. Electro-negative elements should also be tried with a decrease of thermo-electric current looked for upon illumination.

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DIRECT CAPACITY MEASUREMENT

BY

GEORGE A. CAMPBELL

Synopsis: Direct capacity, direct admittance and direct impedance are defined as the branch constants of the particular direct network which is equivalent to any given electrical system. Typical methods of measuring these direct constants are described with especial reference to direct admittance; the substitution alternating current bridge method, due to Colpitts, is the preferred method, and for this suitable variable capacities and conductances are described, and shielding is recommended. Proposed methods are also described involving the introduction of electron tubes into the measuring set, which will reduce the measurement to a single setting or deflection. This gives an alternating current method which is comparable with Maxwell's single null-setting cyclical charge and discharge method. Special attention is drawn to Maxwell's remarkable method which is entirely ignored by at least most of the modern textbooks and handbooks.

The object of this paper is to emphasize the importance of direct capacity networks; to explain various methods of measuring direct capacities; and to advocate the use of the Colpitts substitution method which has been found preëminently satisfactory under the wide range of conditions arising in the communication field.

About thirty years ago telephone engineers substituted the so-called "mutual capacity" measurement for the established "grounded capacity" measurement; this was a distinct advance, since the transmission efficiency is more closely connected with mutual capacity than with grounded capacity. Mutual capacity, however, can give no information respecting crosstalk and accordingly, about twenty years ago, I introduced the measurement of "direct capacity" which enabled us to control crosstalk and to determine more completely how telephone circuits will behave under all possible connections.

For making these direct capacity measurements alternating currents of telephone frequencies were introduced so as to determine more exactly the effective value of the capacity in telephonic transmission, and to include the determination of the associated effective direct conductances which immediately assumed great importance upon the introduction of loading.

Telephone cables and other parts of the telephone plant present the problem of measuring capacities which are quite impossible to isolate, but which must be measured, just as they occur, in association with other capacities; and these associated capacities may be much larger than the particular direct capacity which it is necessary to accurately measure, and have admittances overwhelmingly larger than the direct conductance, which is often the most important quantity. This is the interesting problem of direct capacity measurement, and distinguishes it from ordinary capacity measurements where isolation of the capacity is secured, or at least assumed.

The substitution alternating current bridge method, suggested to me in 1902 by Mr. E. H. Colpitts as a modification of the potentiometer method, has been in general use by us ever since in all cases where accuracy and ease of manipulation are essential.

After first defining direct capacities and describing various methods for measuring them, this paper will explain how this may all be generalized so as to include both the capacity and conductance components of direct admittances, and the inductance and resistance components of direct impedances.[1]

DEFINITION OF DIRECT CAPACITY

It is a familiar fact that two condensers of capacities C_1, C_2, when in parallel or in series, are equivalent to a single capacity (C_1+C_2) or $C_1C_2/(C_1+C_2)$, respectively, directly connecting the two terminals. These equivalent capacities it is proposed to call direct capacities. The rules for determining them may be stated in a form having general applicability, as follows:

Rule 1. The direct capacity which is equivalent to capacities in parallel is equal to their sum.

Rule 2. The direct capacity between two terminals, which is equivalent to two capacities connecting these terminals to a concealed branch-point, is equal to the product of the two capacities divided by the total capacity terminating at the concealed branch-point, i.e., its grounded capacity.

[1] Proofs of the mathematical results included in the present paper will be supplied in an appendix which will be added to the reprint of the paper which is to appear in an early number of the "Bell System Technical Journal."

These rules may be used to determine the direct capacities of any network of condensers, with any number of accessible terminals and any number of concealed branch-points. Thus, all concealed branch-points may be initially considered to be accessible, and they are then eliminated one after another by applying these two rules; the final result is independent of the order in which the points are taken; all may, in fact, be eliminated simul-

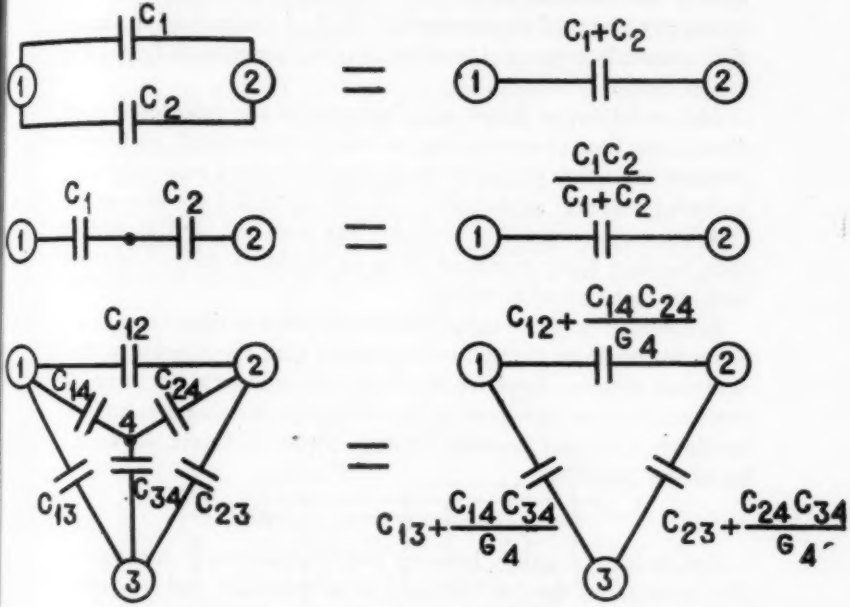

FIG. 1. Equivalent direct capacities. $G_4 = C_{14} + C_{24} + C_{34}$ = grounded capacity of branch-point 4

taneously by means of determinants;—a network of capacities, directly connecting the accessible terminals, without concealed branch-points or capacities in parallel, is the final result. Fig. 1 shows the two elementary cases of direct capacities and also, as an illustration of a more complicated system, the bridge circuit, with three corners 1, 2, 3 assumed to be accessible, and the fourth

inaccessible, or concealed. Generalizing, we have the following definition:

The direct capacities of an electrical system with n given accessible terminals are defined as the $n(n-1)/2$ capacities which, connected between each pair of terminals, will be the exact equivalent of the system in its external reaction upon any other electrical system with which it is associated only by conductive connections through the accessible terminals. The total direct capacity between any group of the terminals and all of the remaining accessible terminals, connected together, is called the grounded capacity of the group.

This definition of direct capacity presents the complete set of direct capacities as constituting an exact, symmetrical, realizable physical substitute for the given electrical system for all purposes, including practical applications. Direct capacities are Maxwell's "coefficients of mutual induction," but with the sign reversed, their number being increased so as to include a direct capacity between each pair of terminals.

In considering direct capacities we exclude any direct coupling, either magnetic or electric, from without with the interior of the electrical system, since we have no concern with its internal structure; we are restricted to its accessible, peripheral points or terminals; some care has been taken to emphasize this in the wording of the definition.

ADDITIVE PROPERTY OF DIRECT CAPACITIES

Connecting a capacity between two terminals adds that capacity to the direct capacity between these terminals, and leaves all other direct capacities unchanged. Connecting the terminals of two distinct electrical systems, in pairs, gives a system in which each direct capacity is the sum of the corresponding two direct capacities in the individual systems. Joining two terminals of a single electrical system to form a single terminal adds together the two direct capacities from the two merged terminals to any third terminal, and leaves all other direct capacities unchanged with the exception of the direct capacity between the two merged terminals, which becomes a short circuit. Combining the termi-

nals into any number of merged groups leaves the total direct capacity between any pair of groups unchanged, and short-circuits all direct capacities within each group.

These several statements of the additive property of direct capacities show the simple manner in which direct capacities are altered under some of the most important external operations which can be made with an electrical network, and explain, in part, the preëminent convenience of direct capacity networks.

Since the additive property of direct capacities is sufficient for explaining the different methods of measuring direct capacities we may now, without further general discussion of direct capacities, proceed to the description of the more important methods of measurement.

COLPITTS SUBSTITUTION BRIDGE METHOD, FIG. 2

The unknown direct capacity is shifted from one side of the bridge to the other, and the balance is restored by adjusting the capacity standard so as to shift back an equal amount of direct

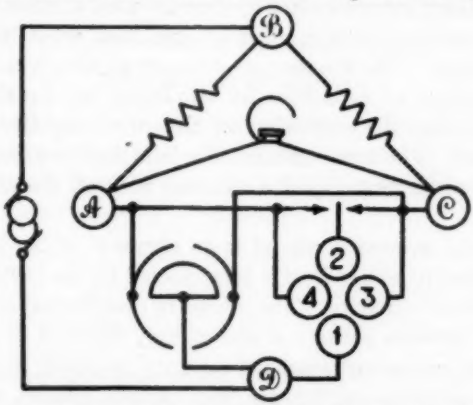

FIG. 2. Colpitts substitution bridge method for direct capacity

capacity. The method is therefore a substitution method, and the value of the bridge ratio is not involved. Both the standard and the unknown remain in the bridge for both settings, so that the method involves transposition rather than simple, ordinary substitution.

Details of the method as shown by Fig. 2 are as follows: To measure the direct capacity C_{12} between terminals 1 and 2 connect one terminal (1) to corner D of the bridge, and adjust for a balance with the other terminal (2) on corner A and then on C, while each and every one of the remaining accessible terminals (3, 4, ...) of the electrical system is permanently connected during the two adjustments to either corner A or C. If the direct capacities in the standard condenser between corners A and D are C', C'' in the two balances,

$$C_{12}=C''-C',$$

and if the bridge ratio is unity,

$$C_{12}-C_{14}=C'+C''-2C^{\circ}$$

where C° is the standard condenser reading when the bridge alone is balanced.

Two settings are required by this method for an individual direct capacity measurement, but in the systematic measurement of all the direct capacities in a system the total number of settings tends to equal the total number of capacities, when this number becomes large. The number of settings may always be kept equal to the number of capacities by employing an equality bridge ratio, and using the expression for the direct capacity difference given above. The same remarks also hold for the group of direct capacities connecting any one terminal with all the other terminals.

In general, ground is placed upon corner C of the bridge, but is transferred to corner D, if it is connected to one terminal of the required direct capacity. The arbitrary distribution of the other terminals between corners A and C may be used to somewhat control the amount of standard capacity required; or it may be helpful in reducing interference from outside sources, when tests are made upon extended circuits. The grounded capacity of a terminal, or group of terminals, is measured by connecting the group to C, and all of the remaining terminals together to D.

The excess of one direct capacity C_{12} over another C_{34} is readily determined by connecting terminals 1 and 5 to corner D, terminals 3, 4, 7, 8, . . . to corner C or A, and then balance with

terminals 2 and 6 on A and C, respectively, and repeat, with their connections reversed.

The required direct capacity C_{12} is balanced against one of its associated direct capacities, augmented by a standard direct capacity C', and the measurement is repeated with the required

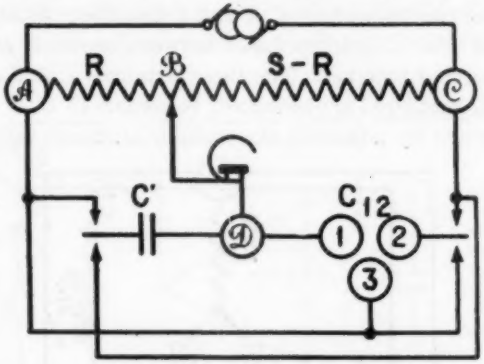

FIG. 3. Potentiometer method for direct capacity

POTENTIOMETER METHOD, FIG. 3

direct capacity and standard interchanged. Let R', R'' be the resistance required in arm AB of the bridge for the first and second balance, then, S being the total slide wire resistance and G_1 the grounded capacity of terminal 1:

$$C_{12} = \frac{R'}{R''}\ C'$$

$$G_1 = \frac{S-R''}{R''}\ C'$$

This ratio method requires for the bridge a variable or slide wire resistance and a constant condenser, and it may be employed as an improvised bridge, when sufficient variable capacity is not available for the Colpitts method. Not being a substitution method, however, greater precautions are necessary for accurate results. There must be no initial direct capacity in arm CD, or a correction will be required. Possibly variable capacity ratio arms would be preferable to resistances.

NULL-IMPEDANCE BRIDGE METHOD FOR DIRECT CAPACITY, FIG. 4

Assuming that the electron tube supplies the means of obtaining an invariable true negative resistance, Fig. 4 shows a method which determines any individual direct capacity from a single bridge setting. The bridge arms are replaced by a Y network made up of two resistances R, R and a negative resistance $-R/2$; the Y has then a null-impedance between corner B and corners A, C connected together. The three terminals 1, 2, 3 of the network to be measured are connected to corners D, C, B and a balance obtained by adjusting the variable standard condenser C'.

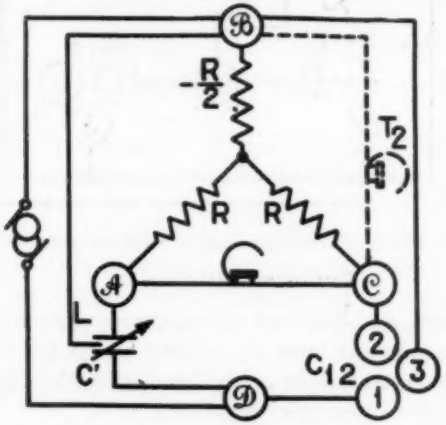

FIG. 4. Null-impedance bridge method for direct capacity

Then $C_{12}=C'$ regardless of the direct capacities associated with C_{12} and C', since these capacities either are short-circuited between corners B, A or B, C or are between corners B, D and thus outside of the bridge.

Correct adjustment of the negative resistance may be checked by observing whether there is silence in telephone T_2 after the balance has been obtained. Assuming invariable negative resistance, this test need be made only when the bridge is set up, or there is a change in frequency. The bridge may be given any ratio Z_1/Z_2 by employing a Y made up of impedances Z_1, Z_2, and $-Z_1Z_2/(Z_1+Z_2)$.

MAXWELL DISCHARGE METHOD,[3] FIG. 5

Connect the terminals between which the direct capacity C_{12} is required, to A, B and the remaining accessible terminals of this electrical system to D. The adjustable standard capacity is C' and any associated direct capacities in this standard are shown as C'', C'''. If C_{12} is a direct capacity to ground, interchange C' and C_{12}. Balancing involves the following repeated cycle of operations:

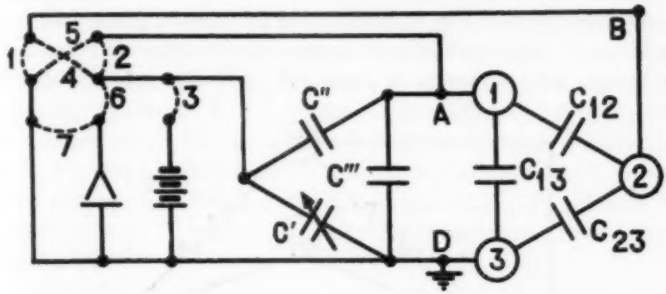

FIG. 5. Maxwell discharge method for direct capacity

1. Make connections 1, 2, 3 and 7 for an instant (thus charging C_{12}, C_{13}, C''', C' and discharging the electrometer).
2. Make connections 4, 5 and then 6 (to discharge condensers C_{13}, C''', mix charges of C_{12}, C' with polarities opposed and connect electrometer).
3. Adjust C' to reduce the electrometer deflection when the cycle is again repeated.

When a null deflection is obtained $C_{12}=C'$; the required direct capacity is equal to the standard direct capacity irrespective of the magnitudes of the four associated direct capacities. If all capacities are free of leakage and absorption, this remarkable method accurately compares two direct capacities by means of a single null setting, and it requires the irreducible minimum amount of apparatus.

[3] Electricity and Magnetism, v. 1, p. 350, (ed. 1892).

BALANCED-TERMINAL CAPACITY MEASUREMENT, FIG. 6

This is defined as the direct capacity between two given terminals with all other terminals left floating and ignored, after a hypothetical redistribution of the total direct capacity from the given pair of terminals to every third terminal which balances the two sides of the pair. The balanced-terminal capacity, as thus defined, is equal to the direct capacity between the pair augmented by one-quarter of the grounded capacity of the pair, neither of which is changed by the assumed method of balancing.

As illustrated in Fig. 6, terminals 1, 2 are the given pair and terminal 3 includes all others, assumed to be connected together. A bridge ratio of unity is employed, and the entire bridge is

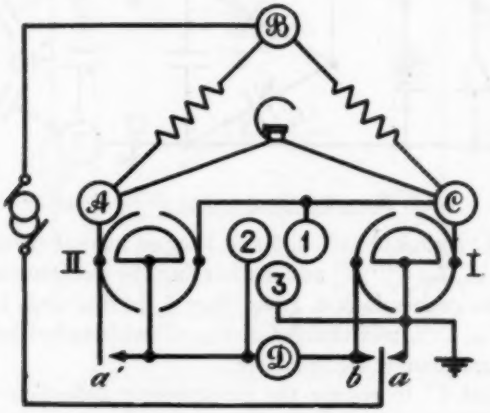

FIG. 6. Bridge for determining hypothetical capacity between two terminals with other terminals balanced and ignored

shielded from ground with the exception of corners C, D which are initially balanced to ground within the range of variable condenser I. The following two successive balances are made:

(1) With contacts a, a' closed and b open, balance is secured by varying condenser I (the total capacity of which is constant) giving the reading C' for its direct capacity in parallel with terminals 1, 3.

(2) With contacts a, a' open and b closed, balance is obtained by varying condenser II, obtaining the reading C'' for its direct capacity in AD.

If C_o', C_o'' are the corresponding readings without the network, the balanced-terminal capacity C_b and the grounded capacity unbalance of the given pair of terminals are:

$$C_b = 2(C'' - C_o'')$$
$$G_2 - G_1 = 2(C' - C_o')$$

Any failure to adjust condenser I to perfectly balance the given pair of terminals, will decrease the measured capacity C_b. This fact may be utilized to measure the capacity with the second bridge arrangement alone, (contacts a, a' open and b closed) by adjusting condenser I so as to make the reading C'' of condenser II a maximum. This procedure presents no difficulty, since the correct setting for condenser I lies midway between its two possible settings for a balance with any given setting of condenser II; furthermore, C'' is not sensitive to small deviations from a true balance in C'.

Balanced-terminal capacity is of practical importance as a measure of the transmission efficiency to be expected from a metallic circuit, if it is subsequently transposed so as to balance it to every other conductor. In practice, when the unbalance of the section of open wire or cable pair, which is being measured, is relatively small, it is sufficient to set condenser I, once for all, to balance the bridge itself and ignore the unbalance of the pair. This favors an unbalanced pair, however, by the amount $(G_2 - G_1)^2/4(G_{12} + G_{CD})$ where $G_{12} + G_{CD}$ is the grounded capacity of the pair augmented by that of the bridge. For rapid working, condenser II is graduated to read $2C''$ and by auxiliary adjustment C_o'' is made zero, so that the required capacity is read directly from the balance.

ADDITIONAL METHODS OF MEASURING DIRECT CAPACITY

Measurement of the capacity between the terminals, taken in pairs with all the remaining terminals left insulated or floating, gives $n(n-1)/2$ independent results, from which all the direct capacities may be derived by calculation of certain determinants.

Practically, however, we are in general interested in determining individual direct capacities from the smallest possible number of measurements, and the first step is naturally to connect all of the remaining conductors together, so as to reduce the system to two direct capacities in addition to the one the value of which is required. Three measurements are then the maximum number required, and we know that two, or even one, is sufficient if particular devices are employed.

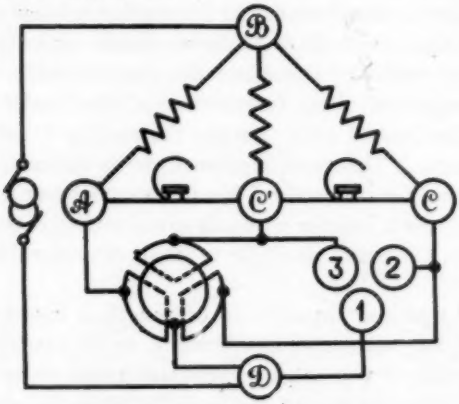

FIG. 7. Double bridge for direct capacity

The three measurement method of determining direct capacities from the grounded capacities of the two terminals taken separately G_1, G_2, and together G_{12}, is given by Maxwell.[3]

$$C_{12}=\tfrac{1}{2}(G_1+G_2-G_{12})$$
$$=\tfrac{1}{2}C'''$$

if $G_1=C'$, $G_{12}=C'+C''$, and $G_2=C''+C'''$; which indicates a method by which large grounded capacities can be balanced against three variable capacities, only one of which need be calibrated, and that one need be no larger than the required direct capacity.

Two-setting methods, as illustrated by the Colpitts and potentiometer methods, rest upon the possibility of connecting one

[3] Ibid., p. 110.

of the associated direct capacities between opposite corners of the bridge where it is without influence on the balance, and not altering any associated direct capacity introduced into the working arms of the bridge. Numerous variations of these methods have been considered which may present advantages under special circumstances. Thus, if conductors 1, 2, 3 of Fig. 7 are in commercial operation, and it is not permissible to directly connect two of them together, a double bridge might be employed with a testing frequency differing from that of operation. A telephone is shown for each ear, and a constant total direct capacity is divided between the three branches in the proportion required to silence both telephones.

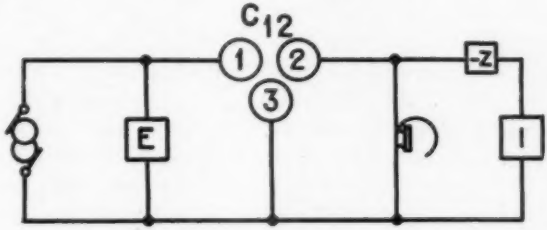

FIG. 8. Ammeter circuit for determining direct capacity

One-setting methods attained ideal simplicity in the Maxwell discharge method, but we found it necessary to use alternating current methods, and here negative resistances make a one-setting method at least theoretically possible, as explained above. Of possible variations it will be sufficient to refer to the ammeter method Fig. 8. Terminals 1 and 2 of the required direct capacity C_{12} are connected to the voltmeter and ammeter terminals, respectively, and all other terminals go to the junction point at 3. Then

$$C_{12}=\frac{I}{2\pi fE}$$

provided the ammeter actually has negligible impedance. The method is well adapted for rapid commercial testing. The ammeter impedance may be reduced to zero by a variable negative

impedance device ($-Z$), adjusted to reduce the shunted telephone to silence.

SHIELDING

In the discussion of the bridge, it has been assumed that the several pieces of apparatus forming the six branches of the bridge have no mutual electrical or magnetic reaction upon each other, except as indicated. In general, however, a balance will be upset by changes in position of the pieces of apparatus, or even by movements of the observer himself, whereas these motions cannot affect any of the mutual reactions which have been explicitly considered. The skillful experimenter, understanding how these variations are produced by the extended electric and magnetic fields, will anticipate this trouble and take the necessary precautions, possibly without slowing down his rate of progress.

Where hundreds of thousands of measurements are to be made, however, substantial savings are effected by arranging the bridge so that reliable measurements can be made by unskilled observers, and here it is necessary to shield the bridge so that any possible movements of the observer and of the apparatus will not affect the results. Magnetic fields of transformers are minimized by using toroidal coils with iron cases. Electrostatic fields are shielded by copper cases; the principles of shielding were explained in an earlier paper,[4] Fig. 13 of this paper showing the complete shielding of the balance as constructed for the measurement of direct capacity by the Colpitts method. Over five million capacity and conductance measurements have been made with the shielded capacity and conductance bridge and in a forthcoming paper Mr. G. A. Anderegg will give details of actual construction of apparatus and of methods of operation as well as some actual representative results.

DIRECT ADMITTANCE MEASUREMENTS

For simplicity, the preceding definitions and methods of measurement have been described in terms of capacity, but everything may be generalized, with minor changes only, for the definition and measurement of direct admittances with their capacity and

[4] The Shielded Balance, El. W., 43, 1904, (647–649).

conductance components. The essential apparatus change is the addition, in parallel with the variable capacity standards employed, of a variable conductance standard, which shifts direct conductance from one side of the bridge to the other, without changing the total reactance and conductance in the two sides of the bridge. This may be practically realized in a great variety of ways as regards details, which it will suffice to illustrate by Fig. 9, where C', C'', C''', G', G'', indicate the continuously variable capacity and conductance standards with enough step-by-step extensions to secure any desired range.

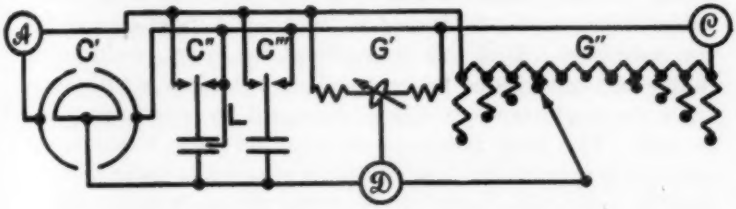

FIG. 9. Variable direct conductance and capacity standard for direct admittance bridge

For the continuously variable conductance standard a slide wire is represented, with a slider made up of two hyperbolic arcs so proportioned that, as the slider is moved uniformly in a given oblique direction, conductance is added uniformly on the left and just enough of the wire is short-circuited to produce an equal conductance decrease on the other side. The arcs are portions of the hyperbola $xy=(L^2-S^2)/4$, where L, S are the total length of the wire and of the portion to be traversed by the slider, and the coordinate axes are the slide wire and the direction of the motion of the slider as oblique asymptotic axes. $L=GS/g=4G/\rho(G^2-g^2)$, where G is the total conductance and $(G\pm g)/2$ the limiting direct conductances on either side.

If an ordinary slider replaces the hyperbolic arc slider, and the scale reading is made non-uniform so as to give one-half of the difference between the direct conductances A to D and C to D, the conductance standard will still give absolutely correct results

with the Colpitts method, provided the bridge ratio is unity. This simplification in connection with the balancing capacity I of Fig. 6 would, however, not be strictly allowable. For improvised testing we have found it sufficient to use two equal resistances (R) with a dial resistance (r) in series with one of them, and take the defect of conductance introduced by the dial resistance as equal to r/R^2 or to $10^{-2}r$, $10^{-1}r$, r, micromho according as R was made 10 000, 3 162, or 1 000 ohms.

For a step-by-step conductance standard, Fig. 9 shows a set of ten equal resistances, connected in series between corners A, C, to the junction points of which there is connected a parabolic fringe of resistances, the largest of which is 2.5 times each of the ten resistances. With this arrangement the direct conductance in AD may be adjusted by ten equal steps, beginning with zero, while the conductance in CD is decreased by equal amounts to zero. The total resistance required for this conductance standard is only 21/25 of the resistance required to make a single isolated conductance equal to one of the ten conductance steps; the ratio may be reduced to $\frac{1}{2}$ by doubling the number of contacts, and using one fringe resistance for all positions. Resistance may be still further economized by using as high a total conductance as is permissible in the bridge, and securing the required shift in conductance from a small central portion of the parabolic fringe.

Fig. 9 shows the variable capacity standards as well as the variable conductance standards and a few practical points connected with the capacity standards may be mentioned here.

The revolving air condenser standard has two fixed plates connected to A and C, so that the capacity will increase as rapidly on one side as it decreases on the other side. Since perfect constancy of the total capacity is not to be expected, on account of lack of perfect mechanical uniformity, the revolving condenser should be calibrated to read one-half of the difference between the capacities on the two sides, as explained above in connection with conductance. The capacity sections employed to extend the range of the revolving condenser include both air condensers

C'' and mica condensers C''', the latter being calibrated by means of the air condensers and the conductance standard.

A novel feature of our standard air condensers is a third terminal called the leakage terminal, and indicated at L in Figs. 4, 9. Attached to it are plates so arranged that all leakages either over, or through, the dielectric supports from either of the two main terminals, must pass to the leakage terminal. There can be no leakage directly from one of the main terminals to the other. There is thus no phase angle defect in the standard direct capacity due to leakage, and that due to dielectric hysteresis in the insulating material is reduced to a negligible amount by extending the leakage plates beyond the dielectric, so as to intercept practically all lines of induction passing through any support. This leakage terminal is connected to corner C of the bridge; in the revolving condensers, it is one of the fixed plates.

DIRECT IMPEDANCE MEASUREMENTS

The reciprocal of a direct admittance is naturally termed a direct impedance; substituting impedance for capacity, the definition of direct capacity, given above, becomes the definition of direct impedance. The complete set of direct impedances constitute an exact, symmetrical, physical substitute for any given electrical system. Direct impedances are often, in whole or in part, the most convenient constants since many electrical networks are made up of, or approximate to, directly connected resistances and inductances. To make direct impedance measurements which will not involve the calculation of reciprocals, we naturally employ inductance and resistance standards in series, the associated direct impedances being eliminated as with direct capacities.

CONCLUSION

It has been necessary to preface the description of methods of measuring direct capacities by definitions and a brief discussion, since direct capacities receive but scant attention in textbooks and handbooks. By presenting direct capacities, direct admittances, and direct impedances as alternative methods of stating the constants of the same direct network, employed as an equiva-

lent substitute for any given electrical system, it is believed the discussion and measurement of networks has been simplified. In another paper the terminology for admittances and impedances will be still further considered, together with their analytical correlation.

DEPARTMENT OF DEVELOPMENT & RESEARCH,
AMERICAN TELEPHONE & TELEGRAPH COMPANY,
JUNE 10, 1922.

NOTICES

Preliminary Report of Committee of the Optical Society of America on the Proposed English Translation of Helmholtz's Physiological Optics

Columbia University, New York, N. Y., June 1, 1922

To the President of the Optical Society of America:

Dear Sir: The Committee appointed by the Council of the Optical Society of America to make arrangements for bringing out an English translation of Helmholtz's "Handbuch der physiologischen Optik" begs to submit the following preliminary report showing the progress of this project up to the present time.

After an extensive canvass of the situation, partly by advertisements in various journals but chiefly by direct correspondence with a large number of prominent scholars and scientists, for example, members of the Optical Society and of the Physical Society, besides psychologists, physiologists, ophthalmologists, oculists, etc., who from different angles and in different degrees might be presumed to be interested in the publication of an English version of this great treatise on physiological optics, the Committee finds that there is a very decided consensus of opinion favorable to going ahead with this enterprise. It is true that a few persons have expressed some doubt as to whether this book which admittedly ranks as one of the classics in science is sufficiently modern to be worth while; but the answer to that objection seemed to be that the third edition at any rate had been brought completely up to date a little more than a decade ago and that it was hardly to be supposed that the permanent value of this authoritative work had been seriously impaired by recent advancements in science, important as these may have been in some instances. However, for this and other reasons the opinion was practically unanimous that the third and, so to speak, definitive edition, edited by Gullstrand, von Kries, and Nagel, and published by Leopold Voss in Leipzig in 1910, was the only one to be considered for reproduction in English.

Accordingly, the Committee proceeded immediately to negotiate with Herr Leopold Voss about obtaining the rights of translation of the third edition in English. He responded to these overtures in a most generous and cordial spirit; and the outcome was that the Committee purchased these rights for the Optical Society at an extremely reasonable price. Herr Voss has also offered to sell the electrotype plates for the engravings and illustrations at a fair figure. The Committee has this offer still under consideration.

Estimates have been obtained from the printers of the cost of manufacturing one thousand copies of the third edition, perhaps in two volumes instead of in three as in the German edition. With additional incidental and unavoidable expenses the total cost will probably amount to ten thousand dollars. The Committee is reliably informed that it is not unreasonable to suppose that about five hundred copies can be disposed of to libraries and institutions in this country and in Great Britain and her dominions. It is fair to assume that an equal number of copies will be sold to individuals, chiefly scholars and professional men, both here and abroad. The retail price of the complete work will be at least $15 per copy. The Committee has received already a little more

than $1000 in advance subscriptions and contributions; and there is at least a possibility of one or two large contributions which may aggregate as much as $1000 more.

As a purely business proposition it might be easily possible to get a reliable firm of publishers to take the work off our hands. Some tentative proposals of this kind have been under consideration. However, the Committee is disposed to think that it will be better for the Optical Society to undertake this business alone and to retain complete control.

In brief the Committee is persuaded that this task can and ought to be brought now to a successful completion, and they have every intention of doing so. Several competent scholars have offered to aid in the work of translation and this work has now been actually begun; but additional helpers are needed, and above all additional contributions are needed. The responsible editors should be chosen without delay. Neither the editors nor their assistants in the work of translation will receive any pecuniary compensation for their labors.

Appended to this report is a complete list of subscribers at the present time. One or two of these subscriptions were for fifty dollars but most of them averaged about fifteen dollars. Anyone subscribing as much as fifteen dollars will be entitled to receive a complete copy of the English edition when published but it is not necessary to send the money in advance. Subscriptions or orders should be sent to Adolph Lomb, Esq., 635 Saint Paul Street, Rochester, N. Y.

The list below contains also the "starred" names of those who are helping to prepare the English translation. The chairman of the Committee would be glad to know of other competent persons who might be willing to contribute in this most serviceable way.

Respectfully,

ADOLPH LOMB,
JAMES P. C. SOUTHALL, *Chairman*,
LEONARD T. TROLAND.

LIST

The following is a complete list of actual subscribers up to June 1, 1922, most of whom have already paid their subscriptions. "Starred" names indicate those who have offered to help with the work of translation.

C. G. Abbot
American Philosophical Society
Adelbert Ames, Jr.
J. A. Andrews
Roswell P. Angier
Wm. L. Benedict
Conrad Berens, Jr.
Wilfrid Blackham
E. V. L. Brown (for Eye Dept., University of Illinois)
Bryn Mawr College Library (C. E. Ferree)
J. McKeen Cattell
Cheney Brothers (per Frank Cheney, jr.)
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John H. Darling
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Western Electric Company Library (per E. H. Calpitto)
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Wisconsin University Library
Claude Wolcott
F. A. Woll
Yale University Library (per James R. Angell)
W. G. Young
Alexander Ziwet
Max Zwillinger

CORRECTIONS

The following changes should be made on page 360 of this volume.

Line Two:

In the expression $\Phi\,(1-\Phi)\,F$ delete the first Φ

Equation 34:

Delete the Φ in the denominator

Equation 36:

Last term should read 19.8 instead of 21.

OPTICAL SOCIETY OF AMERICA

EXHIBIT OF OPTICAL INSTRUMENTS AND APPARATUS NATIONAL BUREAU OF STANDARDS

Oct. 26–28, 1922

Arrangements are now being completed for the exhibit of optical instruments and apparatus to be held at the National Bureau of Standards, Washington, in connection with the annual meeting of the Optical Society of America, Oct. 26–28, 1922.

The leading manufacturers of optical equipment have already signified their intention of participating. However, the exhibit will not be limited to standard commercial types. Individuals and research laboratories are also invited to exhibit special research apparatus. Brief descriptions of instruments and their purposes supplied by the exhibitors will be printed in the program and published later in the minutes of the meeting in the JOURNAL OF THE OPTICAL SOCIETY. The exhibit of new apparatus will thus constitute just as definite a contribution to science as a paper communicated to the meeting. The authors of papers communicated at this meeting are urged to supplement their papers by an exhibit of apparatus in case such an exhibit is suitable and practicable.

Exhibitors are urged to prepare their exhibits and descriptions so as to give them the maximum educational value.

Exhibits must be listed with the committee at the Bureau of Standards not later than Sept. 20, 1922. Blank entry forms for this purpose may be obtained from Prof. C. A. Skinner, Chairman, Exhibit Committee, O. S. A., Bureau of Standards, Washington, D. C. Exhibits may be installed Oct. 24–25 and installation should be completed not later than noon, Oct. 28.

IRWIN G. PRIEST, *Secretary*.